CHAPTER 28

MATHEMATICS

Doctoral Theses

01. AGGARWAL (Anu)
Numerical Solutions of Certain Partial Differential Equations Using Cubic Splines.
Supervisor: Prof. Suruchi Singh
Th 27604

Abstract

The collocation method for the hyperbolic partial differential equations based on cubic splines in the literature is second order accurate. To get high-order accuracy, we present a new scheme that is based on the collocation method using cubic B-splines as interpolating polynomials. We utilize the properties of both cubic B-splines and collocation. In this method, a finite dimensional space is generated by cubic B-splines and the finite uniformly spaced knots in the given domain, which is also called collocation points. Then a solution is chosen which gives a better approximation to the given differential equation. This technique uses a finite difference approximation for temporal discretization and cubic B-splines to interpolate the solution in a spatial direction. In my study, many kinds of hyperbolic partial differential equations are examined. We examined the sine Gordan equation and telegraph in one and higher dimensions. Additionally, we investigated the fractional differential wave equation. For the fractional diffusion equation, we considered the time derivative in Caputo form. We addressed a family of schemes to estimate the solutions of second order hyperbolic partial differential equations subject to Dirichlet and Neumann boundary conditions. These schemes are based on the collocation of cubic B-splines method. The matrix stability method is used to demonstrate the unconditional stability of the scheme. The fourth order accuracy of the scheme is also displayed by considering some already existing examples in literature.

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02. BHARDWAJ (Ruchi)
OPV-Frames and Operator Representation of Frames in Quaternionic Hilbert Spaces.
Supervisor: Prof. Shiv Kumar Kaushik
Th 27606

Abstract

The thesis entitled ``OPV-Frames and Operator Representation of Frames in Quaternionic Hilbert Spaces" deals mainly with the study of operator valued frames (OPV-frames), Gabor frames and frames expressed as operator orbits in quaternionic Hilbert spaces. OPV-frames and their analysis and synthesis operators were defined using tensor products. Based on their properties, OPV-frames are classified as Riesz and orthonormal OPV-frames. Dual OPV-frames and stability of OPV-frames under some perturbation conditions are also discussed. Further, trace-class operators are discussed and some Choi-Kraus type representations of quaternionic quantum channels using OPV-frames are given. Also, Gabor systems for the right quaternionic Hilbert space L^2(R,H) and some operators associated with them are studied and the Walnut`s representation for the Gabor frame operator in quaternionic setting is given. Frames in a right quaternionic Hilbert space that can be expressed as operator orbits under a single generator are discussed and classified the sequences that are expressible as orbit of some operator and approximate frames (that cannot be expressed as orbit of any operator), using the sub-orbit representations of a specified class of operators. Furthermore, the concept of dynamical sampling in quaternionic Hilbert spaces is discussed and the problem to recover the initial state of a function (evolving through an operator) using the set of its spatial samples taken at different time levels is investigated. The mathematical interpretation of the dynamical sampling problem is given. The case of finite and infinite dimensional quaternionic Hilbert space is studied separately and some characterizations for the system of samples to form a frame for a right quaternionic Hilbert space are also given. We discuss general bounded operators in terms of strongly stable contractions and provide some characterizations for the system of spatial samples to form a frame for a right quaternionic Hilbert space.

Contents

03. DEVENDER KUMAR
Dynamical Behaviour of Maps in Topological Dynamical Systems.
Supervisor: Prof. Ruchi Das
Th 27608

Abstract

The thesis explores qualitative aspects of dynamical systems focusing on properties like topological sensitivity, topological equicontinuity, topological pseudo orbit tracing property and open cover specification property in six chapters. Chapter 1 provides preliminaries and historical background. Chapter 2 studies topological sensitivity on semiflows analyzing relations between multi topological sensitivity and thick topological sensitivity. The assertion that a syndetically transitive semiflow on a Urysohn space, possessing a proper compact orbit exhibits syndetic sensitivity is established. Chapter 3 explores topological sensitivities on hyperspace dynamical system. The equivalence between topological asymptotic sensitivity and topological sensitivity for a dynamical system on a compact Hausdorff space is proved. Moreover, it is proved that topological asymptotic sensitivity in the hyperspace dynamical system implies the same for the dynamical system on a compact Hausdorff space. Chapter 4, focusses on topological equicontinuity. It is established that on a first countable, compact Hausdorff space, an almost topologically equicontinuous and

transitive dynamical system is topologically uniformly rigid. Similarly, on a second countable, compact Hausdorff space, any topologically equicontinuous dynamical system with a dense set of periodic points also exhibits topological uniform rigidity. It is also proved that a dynamical system with topological uniform rigidity on a compact Hausdorff space possesses zero topological entropy. Chapter 5 examines the topological pseudo orbit tracing property (POTP). The equivalence between topological sensitivity and cofinite topological sensitivity for dynamical systems on compact Hausdorff spaces is proved and nonzero topological entropy for such systems is shown. Chapter 6 deals with the open cover specification property on topological spaces. It is proved that a dynamical system on a compact Hausdorff space, possessing this property along with the positive orbit expansivity, achieves local eventual onto behaviour. Additionally, the topological entropy of such systems is computed by expressing it in terms of periodic points.

Contents

04. DILIP KUMAR
Some Generalizations of Boas Transforms of Wavelets.
Supervisor: Dr. A. Zothansanga
Th 27609

Abstract

This thesis explores the development and analysis of wavelets through the lens of Boas transforms, extending into fractional realms and integrating with Hartley kernels for enhanced signal processing capabilities. Initially, we introduce fractional Boas transforms, elucidating their properties and associated wavelets, with a particular focus on their vanishing moments. A comparative analysis with the traditional Hilbert transforms establishes a foundation for further innovations in signal analysis. We expand upon the concept of analytic signals proposed by Gabor, proposing generalized analytic signals derived through fractional Boas transforms. This leads to the formulation of generalized analytic wavelets, for which we develop convolution and cross-correlation theorems. Additionally, we define new real wavelets based on the Hartley kernel and Boas transforms, emphasizing their applicability in analyzing asymmetries in real signals. These wavelets are shown to achieve higher vanishing moments and, under certain conditions, form convolution filters with transfer functions that vanish for positive frequencies. Furthermore, we introduce generalized variation diminishing wavelets that exhibit specific sign changes and utilize Tanno's convolution kernels. This comprehensive study not only broadens the theoretical framework of wavelet analysis but also enhances practical applications in signal processing, providing robust tools for the analysis of non-stationary signals and complex data structures.

Contents

05. KOHLI (Teena)
Discrete Time Wilson Frame Systems.
Supervisor: Prof. Tarun Kumar Das
Th 27610

Abstract

This thesis, titled "Discrete Time Wilson Frame Systems" presents a comprehensive exploration of a novel framework in signal processing and its remarkable application in the realm of complex function theory. Specifically, the study focuses on Discrete Time Wilson Frames, their frame operator, perturbation analysis, finite linear combinations, and their ingenious application to solve the classic Littlewood problem within the theory of complex functions. The thesis begins by establishing a solid foundation in frame theory and its extension to discrete-time settings. The concept of Discrete Time Wilson Frames (DTWF) is introduced, providing a bridge between the continuous and discrete domains. The mathematical underpinnings of DTWF are elucidated, including the construction of Wilson frames adapted to discrete signals. The associated frame operator, a key element of frame theory, is defined and analysed in the context of DTWF, revealing its role in signal representation and analysis. The behavior of DTWF in response to perturbations is then investigated, shedding light on the stability and adaptability of these systems for accurate signal reconstruction even under adverse conditions. This analysis not only enhances the theoretical understanding of DTWF but also paves the way for practical applications in noisy signal processing. One of the highlight contributions of this thesis is the exploration of finite linear combinations of DTWF. The formulation of finite linear combinations introduces a new dimension to signal representation, allowing for more concise and tailored descriptions of complex signals. The properties of such combinations are studied, offering insights into their expressive power and computational feasibility. Finally, the thesis addresses the classic Littlewood problem within the theory of complex functions. By formulating the problem in the context of Wilson`s frames, the thesis presents a novel approach to tackle this long-standing problem. The utilization of DTWF facilitates a fresh perspective on the problem and unveils previously undiscovered connections between complex function theory and signal processing techniques. This work opens up new avenues for interdisciplinary research at the crossroads of mathematics, signal processing, and complex function theory.

Contents

06. MAMTA RANI
On the Generalized Class of Primitive and Normal Elements in Finite Fields.
Supervisor: Dr. Sharwan Kumar Tiwari and Dr. Anupama Panigrahi
Th 27613

Abstract

The research work carried out in the thesis deals with the study of the existence of generalized primitive and normal elements, termed r-primitive k-normal, in the extension fields of degree n over the base field of order q (where q is a prime power). We first establish sufficient conditions for the pairs (q, n) which ensure the existence of such elements in the extension fields of degree n over the base field of order q, and

then provide some computational results for particular values of r and k. We provide the complete answer to the existence of 2-primitive 2-normal elements in the fields of odd characteristic. Moreover, we prove that for any q, and any n greater than 6, there always exist 1-primitive 1-normal elements whose reciprocals are also 1-primitive 1-normal. Additionally, for the values of n equals 5 and 6, such elements exist in the fields where q and n are coprime. Moreover, we also extend this problem to the one where the norm and trace of the r-primitive k-normal elements are prescribed. We prove that there always exist r-primitive k-normal elements with prescribed norm and trace in all but finitely many fields, provided n is greater than 4k+5. Furthermore, we investigate the existence of minimal polynomials of such elements with prescribed two coefficients over the base field and prove that such 2-primitive 2-normal polynomials always exist for q and n greater than 10 and 14 respectively. Moreover, for n lying between 9 and 14, and any q, there are 4075 fields where the existence of such polynomials is uncertain.

Contents

07. MAVI (Sneha)
On Key and Abstract Key Polynomials.
Supervisor: Dr. Anuj Bishnoi
Th 27611

Abstract

The research work carried out in the thesis has yielded some significant results on key and abstract key polynomials. In this regard, a connection between MacLane-Vaquié chains and complete sequences of abstract key polynomials is obtained. Some applications of these two notions are also explored. For instance, using MacLane-Vaquié chains a characterization of immediate approximation types, valuation-extending approximation types and almost pure extensions is given. Further, some precise conditions are obtained under which a valuation-transcendental extension is induced by a pseudo-monotone sequence. Finally, for a henselian valued field, a characterization of liftings of polynomials, distinguished pairs and saturated distinguished chains using abstract key polynomials are also given.

Contents

08. PANCHAL (Rahul)
Extrapolation and Weighted Norm Inequalities in Lebesgue Type Spaces.
Supervisor: Prof. Arun Pal Singh
Th 27895

Abstract

The Rubio de Francia extrapolation theorem is a fundamental result in harmonic analysis that allows for the extension of weighted norm inequalities from one exponent to a whole range of exponent values. This theorem was introduced by J. L. Rubio de Francia. This theorem states that If a sublinear operator T is bounded on $L_{p_0}^w$ for some p_0, where w is a weight in Muckenhoupt class A_{p_0} then T is also bounded on L_p^w for any p in a rangeand every weight w in A_p . This extrapolation theorem was re-investigated and explored by many people. In this thesis, we study rubio de Francia extrapolation theorem in variable Lebesgue spaces with B_p(.) weight class. The case of B_∞ weights has also been proved and as applications we give a weighted norm inequality and we extrapolate a vector valued inequality. We also prove extrapolation theorems in Lebesgue and grand Lebesgue spaces for quasi monotone functions. As an application we characterize the boundedness of hardy averaging operator on grand Lebesgue spaces for quasi monotone functions. We prove that the quasi-grand Lebesgue space (q-GLS) $G\psi_{a,b}$ is not normable and prove some weighted inequalities involving Hausdorff and Dunkl-Hausdorff operator for some particular value of $\psi.$ finally, we investigate a weighted inequality for discrete generalized Hardy averaging operator. In this connection, we define a new weight class $\mathcal{QB}_{\beta,p}$ and study some of its properties.

Contents

09. PANDEY (Ashutosh)
Boundary Value Problems on Certain H-type Groups.
Supervisor: Dr. Mukund Madhav Mishra
Th 27620

Abstract

In this thesis, we have analysed the Neumann problem for various domains in groups of Heisenberg type. We also study the polyharmonic Neumann BVPs and polyharmonic mixed BVPs. The outline of different chapters of this thesis is as follows. Chapter 1 presents the heuristics needed for the smooth reading of the rest of the thesis. In Chapter 2, we start with a model unbounded domain, namely the upper half-space Ω = {(ζ , t) ∈ Hn : t > 0} and analyse the following Neumann problem ΔHnu=0 inΩ, ∂ u = g on ∂Ω, ∂n where n denote the outward unit normal at the boundary ∂Ω. We then propose an additional condition on the boundary data so that the integrals converge near infinity and hence obtain the necessary and sufficient conditions for the solvability of the Neumann problem. Later we construct a Green's type function that solves the associated inhomogeneous problem for a circular data. In Chapter 3, the domain under consideration is an infinite strip in Hn. Here we first construct a Neumann function and propose an explicit representation formula for the solution of a Neumann BVP. Later using this representation, the existence of the solution is established. In chapter 4, we discuss the solvability of the polyharmonic Neumann problem, polyharmonic mixed BVPs, the Neumann-Dirichlet and Dirichlet-Neumann problem on the half-space in Hn. Chapter 5 is an effort to open the discussion of Neumann BVP for subelliptic operators on a more general class of step two nipotent Lie groups. Here, we present the case of a homogeneous Neumann BVP

in an H-type group. The results establish that similar results are possible in this set up.

Contents

10. SATISH KUMAR
Topological Approaches to Vector Variational Inequalities.
Supervisor: Prof. Pankaj Kumar Garg
Th 27614

Abstract

The present thesis focuses on the study of vector variational inequality problems and their generalized forms, generalized vector equilibrium problems, and generalized vector saddle point problems. In the existing literature, these problems are mostly found to have solutions by using several kinds of monotonicity, convexity, etc. of the function involved. But here we provide solutions to these problems by using a significantly different approach (topological approach) from the existing ones. In our method, we use several concepts of topology such as closedness, compactness, upper semi-continuity, lower semi-continuity, net theoretic convergence, and admissibility of function space topology, etc. The admissibility of function space topology is a major tool to obtain our results. Apart from this, the KKM-Theorem is being used frequently throughout the thesis. In recent years, topology has found popularity from researchers working not only in the field of mathematics but also in various diverse fields such as economics, genetics, engineering and computer science etc. Nada and Zohny investigated the development process of an embryo from zygote until birth utilizing topological dynamical system (which is a branch of dynamical system) and they made three conjectures related to it. Recently, Acharjee et al. introduced the notion of a bi-topological dynamical system and they disproved the conjectures of Nada and Zohny. Topological methods are also being applied to provide solutions pertaining to optimization theory, game theory, etc. These studies have motivated us to study and apply topological methods to solve various types of variational inequality problems and other similar problems. They include VVIP and its generalized forms, equilibrium problems, and saddle point problems. The investigation provided in our thesis establishes topological approaches as effective and successful ones for solving variational inequalities and similar problems.

Contents

11. SATYAPRIYA

Frame Multiresolution Analysis and Associated Wavelet Frames on Locally Compact Abelian Groups.

Supervisor: Prof. Raj Kumar

Th 27615

Abstract

In the course of this thesis, an exhaustive examination of the construction process of a frame multiresolution analysis (FMRA) on a locally compact Abelian group has been undertaken. The study delves into the intricacies of establishing an effective FMRA, addressing various scenarios where achieving the desired FMRA proves challenging. In instances where the expected FMRA is elusive, a necessity arises to alter the approach. The thesis elucidates the systematic construction of wavelet frames, including multiwavelet frames, derived from the established FMRA. Special attention is dedicated to the investigation of cases involving dyadic dilations, presenting a focused analysis of the conditions under which a wavelet function alone proves sufficient for generating a comprehensive wavelet frame. Throughout the exploration of each construction process, the thesis incorporates detailed illustrative examples to enhance understanding and facilitate a more profound grasp of the concepts presented. By dissecting both successful and challenging instances, the research not only contributes to the theoretical foundation of frame multiresolution analysis but also provides practical insights into adapting methodologies when faced with obstacles. The inclusion of comprehensive examples serves to bridge the theoretical framework with real-world applications, promoting a more holistic comprehension of the subject matter.

Contents

12. SHARMA (Amit)

Adaptive Numerical Techniques for Singularly Perturbed Differential Equations with Delay.

Supervisor: Dr. Pratima Rai

Th 27616

Abstract

The present study is devoted to the development of higher-order uniformly convergent numerical methods for solving different classes of convection-diffusion singularly perturbed ordinary/partial differential equations with large delay. We consider the class of singularly perturbed delay differential equations (SPDDEs) with and without turning points and the class of SPDDEs with integral boundary conditions for their numerical approximation. Due to the presence of a delay, the exact solutions of the problems possess interior layer along with the boundary layer(s). Error analysis is performed to establish the consistency, stability and parameter-uniform convergence for the proposed numerical methods. Numerical experiments are conducted for validation of the theoretical findings and demonstration of the efficiency of the proposed techniques.

Contents

13. SHARMA (Nitin)
Woven and Fusion Frames in Quaternionic Hilbert Spaces.
Supervisor: Dr. Khole Timothy Poumai
Th 27618

Abstract

The thesis entitled “Woven and Fusion Frames in Quaternionic Hilbert Spaces” deals with the study of various kinds of woven frames in quaternionic Hilbert spaces. Several types of woven frames have been introduced and studied in quaternionic Hilbert spaces. A characterization of weaving frames is given and several results related to the construction of new woven frames from existing woven frames in quaternionic Hilbert spaces have been proved. Also, a necessary and sufficient condition for a family of subspaces to be a fusion frame, in terms of the synthesis operator, is proved. Further, various types of perturbation of fusion frames in quaternionic Hilbert spaces have been discussed and studied. Woven fusion frame has been defined in a quaternionic Hilbert space and a necessary condition under which two families of fusion frames forms a woven fusion frame is obtained. Woven K-frames in quaternionic Hilbert spaces are introduced and a necessary and sufficient condition for a family of K-frames to be a woven K-frame is given. Also, some conditions under which the bounded image of a woven K-frame is woven K-frame are given. A perturbation result for woven K-frame in a quaternionic Hilbert space is proved. Some properties of K-fusion frames in quaternionic Hilbert spaces are also studied. It has been proved that, under certain conditions, a family of Bessel sequences can produce a K-fusion frame. A necessary and sufficient condition for the bounded image of a K-fusion frame to form a K-fusion frame is given. Further, a Paley Wiener type perturbation result for K-fusion frame has been examined and a necessary and sufficient condition under which a family of Bessel sequences forms a woven K-fusion frame is proved. Finally, a necessary and sufficient condition under which a family of K-fusion frames forms a woven K-fusion frame is obtained.

Contents

14. SHARMA (Avnish Kumar)
Existence of Pairs of Primitive and Normal Elements over Finite Fields.
Supervisors: Dr. Sharwan Kumar Tiwari and Dr. Anuj Bishnoi
Th 27617

Abstract

The research work carried out in the thesis deals with the study of the existence of pairs (a, b) in the extension fields of degree n over the base field, a finite field of order q (where q is a prime power), such that both a and b are primitive as well as normal, and b is the value of polynomials or rational functions with non-constant denominators, satisfying certain conditions, at the element a. We first establish sufficient conditions for this existence, and then provide some computational results for particular types of polynomials and rational functions. We show that for the fields of characteristic 3 and quadratic polynomials with non-zero discriminants, there are at most 21 fields in which such a pair may not exist, and only 5 of these are genuine exceptions. Moreover, for the fields of even characteristic and rational functions with non-constant denominators, we obtain that there are only 55 fields in which the existence of such a pair is not guaranteed. Furthermore, we also extend this problem to the one where the norm and trace of the element a are prescribed. In particular, for the fields of characteristic 7 and quadratic polynomials, we find that if the degree of the extension field is greater than or equal to 8, the desired pairs exist in all but 5 possible fields. In the case of rational functions, considering fields with characteristic 11, we identify that there are only 2 possible fields in which the existence of the desired pair is not guaranteed.

Contents

15. SINGH (Beenu)
Variants of Connectedness in Proximity and *S*-Proximity Spaces.
Supervisor: Dr. Davinder Singh
Th 27619

Abstract

A proximity space is a generalization of a metric space and a topological group. The motivation for the axioms of proximity discovered by Efremovic in 1951 comes from the gap or distance between two sets in a metric space. Connectedness plays an important role in topology. The notion of connectedness in proximity spaces was introduced by Mrowka and Pervin. The aim of this thesis is to study various types of connectedness in proximity spaces. It consists of five chapters. The chapter-wise sketch is as follows: In Chapter 1, we discusses historical developments, basic notions and results of proximity and S-proximity spaces and their connectedness. Chapter 2 is devoted to the notion of sum δ-connected proximity spaces. The purpose of Chapter 3 is to deal with the concept of S-δ-connectedness in S-proximity spaces. We are concerned with the notion of δ^*-connectedness by using ideal in proximity spaces that is analogous to the notion of *s-connectedness in ideal topological spaces in Chapter 4. In Chapter 5, we introduce the notion of δ-connectedness modulo an ideal in proximity spaces by utilizing an ideal which naturally extends δ-connectedness in its usual sense.

Contents

16. SUNDER DEEP
Recovery Analysis of Sparse Signals Through Greedy Algorithms.
Supervisors: Dr. Shiv K. Kaushik and Dr. Khole Timothy Paomai
Th 27607

Abstract

The primary goal of the thesis is to give sufficient conditions for the recovery of sparse vectors through greedy algorithms using frames and g-frames. We divide the thesis into five Chapters. Chapter 1 thoroughly introduces the fundamentals of signal processing and the theory of frames in Hilbert space to make the thesis self-contained. In Chapter 2, we study Lebesgue-type inequalities to acquire upper estimates for the errors and convergence of the block orthogonal greedy algorithm (BOGA) with regard to g-frames with small coherence. Chapter 3 deals with recovering block sparse signals from non-uniform finite wavelet frame transform (NUFWFT). In Chapter 4, we obtain a sufficient condition for BOGA to exactly recover the block sparse solution via block RIP inherited from g-frames. In Chapter 5, the Tikhonov Orthogonal Greedy Algorithm (T-OGA), a variant of the orthogonal greedy algorithm (OGA), is introduced. The motive behind defining T-OGA is to recover the sparse signal from the noisy measurements efficiently.

Contents

17. YADAV (Ram Prasad)
Discontinuous Galerkin Finite Element Method for Certain Singularly Perturbed Problems with Discontinuous Data.
Supervisor: Dr. Pratima Rai
Th 27621

Abstract

The thesis addresses the numerical approximation of singularly perturbed problems with discontinuous data using the discontinuous Galerkin finite element method`s non-symmetric interior penalty Galerkin variant. It systematically addresses various classes of SPPs with discontinuities in the coefficients and the source function, providing both theoretical analysis and numerical validation of the proposed methods. By establishing uniform convergence and error estimates in both energy and balanced norms, this study comprehensively evaluates the NIPG scheme`s effectiveness in approximating the considered class of problems.

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Discontinuous Reaction Coefficient and Source Function 6. Singularly Perturbed Parabolic Problems with Discontinuous Convection and Reaction Coefficients, and Source Function 7. Singularly Perturbed Parabolic Problems with Discontinuous Diffusion, Reaction Coefficient, and Source Function. Conclusion and Future Work. List of Publications. References.