CHAPTER 28

MATHEMATICS

Doctoral Theses

01. ABDAL (Syed Mohammad)
Approximate Controllability of Semilinear Control Systems with Delay and Impulses.
Supervisor: Dr. Surendra Kumar
Th 26093

Abstract

In control theory, controllability is a qualitative property of a dynamical system. Whereas, approximate controllability is one of the most prominent concepts of controllability for infinite dimensional differential equations, and the research for it is of great significance. Almost all the work of our thesis revolves around the study of approximate controllability of semilinear control systems or semilinear measure driven control systems involving infinite delay or state-dependent delay (SDD) in Hilbert spaces. In a delayed system both the present and past states affect the future states of the system. Functional differential equations having bounded or unbounded delay are abstract formulation of several mathematical models arise in a large number of physical, chemical or biological processes. Many real life problems such as reaction diffusion, two-body problem in classical electrodynamics, and many more are modeled as differential equations involving SDD [20, Chapter 5]. Together with these control systems, we also included and discussed theory of variable delay in control, instantaneous impulse, non-instantaneous impulse, nonlocal initial conditions in different chapters. Also, approximate controllability of a dynamical system studied where instantaneous and non-instantaneous impulses are included together for the first time. In this thesis, we use theories of semigroup operators, evolution operators, resolvent operators, fractional power operators, regulated functions, Lebesgue–Stieltjes integrals as well as Hausdorff measure of non-compactness. Different fixed point theorems are applied to prove the existence of a mild solution. To be more concise, we explore some sufficient conditions in proving approximate controllability. Main technique in this thesis is founded on the fixed point theory and trajectory reachable set approach.

Contents

02. ANEJA (Neetu)
Synchronization Methods for Chaotic and Hyperchaotic Systems.
Supervisor: Prof. Binay Kumar Sharma
Th 26104

Abstract

"The three words Non-linearity, Chaos and Synchronization are closely related to our life. We often face non-linearity and chaoticity in our daily life and we need synchronization at almost every step of life. Chaotic behaviour exists in systems with nonlinear dynamics. Synchronization is the coordination of event to operate a system in unison. Thus, chaos synchronization simply means the study of synchronization in chaotic systems and it is the most popular phenomenon which can be understood within the unifying framework of the non-linear sciences. The investigations of different synchronization techniques have gained much importance in last decade due to its vast applications in secure communication, parammeter estimation and chaos anticipation. The main objective of this thesis is to investigate different synchronization methods for chaotic and hyperchaotic systems. Also, we introduced Multiswitching Synchronization for fractional order chaotic systems with different order of derivative. Chapter 1 is the introductory chapter. It gives the brief survey of the history and basic results related to synchronization of chaotic systems. Chapter 2 entitled "Hybrid Projective Synchronization of A New Hyper Chaotic System" gives an insight for important tools to study the stability of dynamical behavior of newly constructed hyper chaotic systems within some certain parameter range. In Chapter 3 entitled "Multiswitching Synchronization of Fractional Orders Chaotic Systems with different order" explains multiswitching synchronization scheme for a class of fractional order chaotic systems by combining active and adaptive control theories. Adaptive controllers have been designed by using different laws of switching and fractional order Lyapunov stability theory has been discussed. We have also constructed a new fractional order duffing system. Chapter 4 entitled "Synchronization with Disturbance of a Fractional Order Hyperchaotic System" provides an insight for dynamical behaviour of a fractional order hyper chaotic system via some analytical and quantitative approach. Analytically, we have drawn Poincar ´ e surface of section and chaotic attractors using Mathematica PREFACE and Matlab. In Chapter 5 entitled "Reduced Order Multiswitching Synchronization" we have investigated the problem of reduced order multiswitching synchronization using active control method. Reduced order multi-switching synchronization can be considered as a combination of multiswitching synchronization and reduced order synchronization.

Contents

03. ANURAG KUMAR
Third and Fifth Order Finite Difference WENO Schemes for Hyperbolic Conservation Laws.
Supervisor: Dr. Bhavneet Kaur
Th 26105

Abstract

This thesis presents various improved weighted essentially non-oscillatory (WENO) schemes to approximate the solutions of the multi-dimensional hyperbolic conservation laws, especially, focusing on third and fifth order finite difference WENO schemes. We modify the non-linear weight functions wk by using the power parameter 'p' which is one of the most important characteristic of the WENO

schemes. We use this parameter on un-normalized weights (αk) as the power of global smoothness indicator for each proposed WENO scheme. This parameter helps to balance the order of accuracy of the overall WENO scheme. We satisfy the sufficient conditions at the critical points for each third and fifth order WENO schemes. The sufficient condition indicates the convergence of the non-linear weights to the linear weights for the WENO scheme. The performance of the proposed WENO schemes is compared with the previously developed WENO schemes in terms of resolution, efficiency and order of accuracy by solving several one- and two-dimensional shock capturing test problems. The chapterwise description of the thesis is as follows: Chapter 1 contains the basic terminology and results to be used in the subsequent chapters. We have discussed the general mathematical reconstruction procedure of the finite difference WENO scheme which is used in the entire thesis Chapter 2 deals with the construction of a new variant of the third order WENOZ3 scheme based on a simple global smoothness indicator that satisfies the sufficient condition for convergence in terms of the non-linear weights. By using this simple global smoothness indicator, the proposed WENO-MES3 scheme provides the third order accuracy and also reduces the computational time at the critical points. Chapter 3 deals with a new improvement of third order weighted essentially non-oscillatory scheme (WENO-AB3) in the finite difference framework for hyperbolic conservation laws In Chapter 4, we propose an extended variant of the third order finite difference WENO-L3 scheme to upgrade the rate of convergence and resolutions for the hyperbolic conservation laws. The proposed WENO scheme is abbreviated as WENO-L3+ scheme. The WENO-L3+ scheme uses three points data S3{xi−1, xi, xi+1} same as other third order WENO schemes. Chapter 5 deals with a new fifth order finite difference WENO-NZ5 scheme with less dissipation to approximate the solutions for one- and two-dimensional hyperbolic conservation laws and associated problems.

Contents

04. ASHOK KUMAR
Transportation and Transshipment Problems in Intuitionistic Fuzzy and Neutrosophic Environment.
Supervisor: Prof. Ratnesh Rajan Saxena
Th 26102

Abstract

A transportation problem (TP), is a special class of linear programming problems (LPPs) that aims at selecting the most economical shipping paths for the displacement of a homogeneous quantity from several origins/sources to various destinations. A special type of transportation problem is known as a trans- shipment problem in which goods are transported from a source to a destination through various intermediate nodes (sources/destinations), possibly to change the modes of transportation or consolidation of smaller shipments into larger or de- consolidation of shipments. These problems have found great applications in the era of e-commerce. The formulation of transportation/transshipment problems involves knowledge of parameters like supply, demand, related cost, time, ware- house space,

budget, etc. In order to model such problems, imprecise data is best represented as intuitionistic fuzzy numbers. Intuitionistic fuzzy number(IFN) encompasses imprecision due to vagueness and ambiguity thereby taking into ac- count the hesitation in defining the related cost coefficients and parameters in a fuzzy environment. The generalization of fuzzy and intuitionistic fuzzy sets, known as neutrosophic sets, has been introduced to express uncertain, incomplete, and indeterminate knowledge encountered in a real-world problem. Apart from the uncertainty or vagueness of the parameters of the transportation/transshipment problem in real life, there is some indeterminacy due to various reasons, such as imperfection of the data, ignorance of the problem, poor status forecasting, etc. Thus, to handle such problems amicably, transportation/transshipment problems are modelled in a neutrosophic environment.

Contents

1. Introduction 2. Intutionistic fuzzy transshipment problem 3. Intutionistic fuzzy transshipment problem 4. Neutrosophic fuzzy transportation problem 5. Neutrosophic fuzzy transshipment problem and References.

05. BASUMATARY (Lakshmi Rani)

Heat and Mass Transfer Analysis of Free and Forced Convection in Horizontal Rectangular Enclosure With Wall Heat and Concentration Sources.

Supervisor: Prof. Vusala Ambethkar

Th 26109

Abstract

Fluid ow with heat and mass transfer in a horizontal rectanglular enclosure plays a very important role in nature, living organisms and a variety of practical situations. The present thesis investigates the heat and mass transfer analy- sis of free and forced convection in horizontal rectangular enclosure with wall heat and ncentration sources. It deals with the numerical solutions of two dimensional steady-state ncompressible uid ow with heat and mass trans-fer analysis in a horizontal rectangular enclosure using nite volume method. Finite volume method (FVM) is the numerical discretization technique which transforms partial di erential equations like Navier-Stokes equation into alge- braic equations over control volume. In the rst step of solution processis the discretization of geometric domain. In FVM, volume integrals in partial dif- ferential equation, which contains a divergent term are converted to surface integrals by using Gauss's divergence theorem. The uid ow with heat and mass transfer analysis of free and forced con- vection in horizontal rectangular enclosure at low, moderate and high Reynolds numbers by taking di erent Prandtl, Schmidt, Richardson, Rayleigh numbers along with slip and no-slip wall boundary conditions has remained an area of considerable challenge. The importance of this research is that the problems discussed in this research work illustrates \internal ows" and internal ows are complex and more signi cant than the "external ows". Motivation for this thesis is due to wide range of applications in engineering and industry. There are so many applications includes the application of heat and mass transfer respectively. Our goal in this thesis is to analyze both heat and mass transfer of free and forced convective ows in rectangular enclosure with wall heat and concentration sources as well. The thesis comprises of six chapters.

Contents

1. General introduction 2. Heat transfer analysis of forced convective flow in horizontal rectangular enclosure with wall heat source 3. Mass transfer analysis of free convective flow in horizontal rectangular enclosure with wall concentration source 4. Free convective flow in horizontal rectangular enclosure with wall heat and

concentration sources 5. Combined convective flow in horizontal rectangular enclosure with wall heat source 6. Heat and mass transfer analysis of combined convection in horizontal rectangular enclosure with wall heat and concentration sources. Further research. Appendix A: Pseudocodes B: Code Validation and References.

06. BHATT (Sandeep)
Numerical Solutions for Certain Parabolic Differential Equations Using Cubic B-Spline Techniques.
Supervisor: Prof. Swarn Singh
Th 26096

Abstract

Parabolic differential equations arise in many areas of science and engineering. Some famous examples of such differential equations are Burger's equation, heat equation, convection-diffusion equation, Fisher's reaction-diffusion equations, generalized Burger- Fisher equation and etc. These equations have application in areas of gas dynamics, heat conduction, traffic flow, fluid mechanics, population dynamics etc. Researchers have been developing. Researchers have been developing a lot of methods to find the solutions of these equations because of their applications in real life scenario. There are various problems that can be solved analytically but there are also problems whose analytical solution is not that easy to find. This is where the need of numerical methods arise. Numerical methods are easy to implement and the solutions approximated by these methods are very close to the analytical solutions. In the past various finite difference methods [2,3,6,8,11,19,20,24] had been discussed to find the solutions of these problems. In finite difference method, we descritize the domain into grid points and use finite difference approximation of the derivatives to approximate the solution at grid points. These methods does not provide any information regarding the solution at points of the domain other than grid points. Collocation methods on the other hand provide the approximation of the solution throughout the domain. Most popular collocation methods are based on splines and B-spline. Splines are piecewise polynomials that have high degree of smoothness at the connecting points of polynomial and B-splines are the basis for the space of spline functions. A lot of collocation and B-spline collocations have been discussed [1, 7, 10, 12–14, 16, 21–23] to solve parabolic differential equations. Many researchers have discussed collocation methods based on splines such as cubic, quadratic, quintic, etc. As we increase the degree of splines, we get methods with higher order of accuracy but these methods require a lot of computation time. The primary objective of this thesis is to develop numerical methods to solve linear and non-linear parabolic differential equation by modifying the already existed cubic B-spline collocation methods. We discuss methods based on finite difference and collocation of cubic B-splines to approximate solutions of linear and non-linear parabolic differential equations over uniform and non-uniform mesh subject to Dirichlet, Neumann and mixed boundary conditions. We have also discussed the stability of the proposed methods. This thesis consists of seven chapters followed by a section presenting the scope for future research work and a list of references useful for the development and application of the methods discussed.

Contents

1. Introduction 2. Cubic B-Splines on non-uniform mesh for solving non-linear parabolic differential equation 3. Fourth order accurate cubic B-Spline collocation method for linear parabolic differential equation 4. Numerical solution of convection

diffusion equation using optimal cubic spline method 5. Numerical method for solving fisher's equation using cubic B-Splines 6. Unconditionally stable cubic B-Spline collocation method for the solution of fourth order parabolic differential equation 7. Summary. Scope for Future Research work and Bibliography.

07. DHARMENDRA KUMAR
Conservativity, Stability and Perturbations of Quantum Dynamical Semigroups.
Supervisor: Prof. Sachi Srivastava
Th 26095

Abstract

Markov (or semi-Markov) semigroups occur naturally in the description of evolution of probability measures, with Markov processes, or by duality, in the description of evolution in the space of functions on the relevant state space. Quantum dynamical semigroups (QDS) provides us a framework for mathematical description of the irreversible dynamics of open quantum sys- tems. By an open quantum system we mean a quantum-mechanical system that interacts with external environment. Particularly in non-communicative case, completely positive maps (one of the dening properties of QDS) play an important role in the description of open quantum system. Beginning with Feller [6] and Yosida [15], many authors (Kato [7], Davies [3], Goswami-Sinha [12], Fagnola [5], Sinha-Srivastava [13]) have described the construction and properties of the minimal semigroup, associated with the Chapman-Kolmogoro equation or equivalently its predual version, that is, the Fokker-Planck equation. For an introduction, one may refer to [3], [12, Chapter 3] and [13, Chapter 6]. These are special C0-semigroups of com- pletely positive (CP) maps on B(H), the C-(or von Neumann) algebra of all bounded linear operators on a Hilbert space H. In the earlier works, in the context of classical probability, the -algebra considered was the com- mutative algebra of complex-valued function on the state (or sample) space, thereby making the property of positivity and complete positivity equiva- lent, and hence leading to study and construction of Markov (positivity and identity-preserving) semigroups. The transition to the non-commutative or Quantum picture happens when the state space metamorphoses into a suit- able Hilbert space H, the probability measures to the \density-matrices " or the non-negative trace-class operators in H of trace 1, while the \space of functions on the state space" to B(H): In such a case, a special C0-semigroup, say, T = (Tt)t0 describes the evolution in the space of trace-class operators and its dual W = (Wt)t0 ; often called the Quantum Dynamical Semigroup and abbreviated as QDS, describes that of the \functions " in B(H). For an introduction into these ideas, we refer to [9]. In this thesis we study some aspects of QDS like perturbations of two minimal QDSs, stability of QDS and quantum extension through conserva-tivity. Let H be a separable complex Hilbert space.

Contents

08. DINESH KUMAR
Restricted Problem of 2+2 Bodies with Straight Segment Under the Effect of Oblateness, Radiation Pressure, Coriolis and Centrifugal Forces.
Supervisor: Prof. Rajiv Aggarwal
Th 26106

Abstract

In celestial mechanics, restricted three-body problem is the most fundamental and well-known problem. This problem has been studied in depth by a large number of researchers and scientists over the years. In this problem, two bodies revolve around their center of mass in circular orbits under the influence of their mutual gravitational attraction and a third body (influenced by the previous two but not influencing their motion) moves in the plane defned by the two revolving bodies. The restricted three- body problem is to describe the motion of the third body (Szebehely (1967)). Whipple and Szebehely (1984) were the first to generalize the restricted three-body problem to the problem of n+v bodies. They deduced the system's integral for n = 2 and v 1 by using the gravitational forces. A simpler form of the restricted problem of n+v bodies was explored by Whipple (1984) by taking the value of n = v = 2 into consideration. He investigated the existence and stability of the equilibrium points of two infinitesimal bodies, which move under the gravitational field of two massive bodies and their mutual gravitational attraction. He determined that there exist fourteen equilibrium points of the restricted problem of 2 + 2 bodies out of which six are collinear with the centers of the massive bodies and eight are non-collinear. Kalvouridis and Mavraganis (1995) studied the equilibrium points and their stability of the restricted problem of 2+2 bodies by adding the radiation pressure effect of the two primaries. In this continuation, the combined effects of photogravitational and oblateness on the equilibrium points and regions of motion of the in nitesimal bodies were deliberated by Kalvouridis (1997). An extension of the Robe's restricted three-body problem (Robe (1977)) to the Robe's restricted problem of 2 + 2 bodies initially studied by Kaur and Aggarwal (2012). They considered two infinitesimal bodies inside the more massive primary and studied the existence and stability of the equilibrium points. They determined four collinear and infinite number of non-collinear equilibrium points. They also concluded that all the equilibrium points are unstable for all the values of the involved parameters.

Contents

09. GUPTA (Prachi)
Radius Constants and Differential Subordination for Certain Subclasses of Analytic Functions.
Supervisors: Dr. Sumit Nagpal and Prof. V. Ravichandran
<u>Th 26098</u>

Contents

10. JAIN (Shikha)
Qualitative Study and Epidemiological Modeling of Certain Complex Realities.
Supervisor: Dr. Sachin Kumar
Th 26094

Abstract

Mathematical modeling is the art of transforming real-world problems into tractable mathematical equations whose theoretical and numerical analysis gives insight, solutions, and guidance helpful for the originating problem. Therefore, it plays a significant role in our day-to-day life. Mathematical models are advantageous tools for building and testing theories, for a thorough understanding of the modeled systems, answering specific questions, giving direction for problem solving, studying the e effects of different components, and making predictions [16]. It is further bifurcated into various interesting branches such as epidemiological modeling, ecological modeling, psychological modeling, and many others. In this thesis, we are going to study various epidemiological compartment models. Communicable diseases have always been a significant segment of human life [15]. In the past, there have been various epidemics that have invaded the human population and caused many deaths before disappearing. Sometimes they have recurrent outbreaks with small gaps and persist in the population while sometimes population develops immunity against them and they no longer remain a threat to humans. Modeling of infectious diseases is very helpful in understanding how infectious diseases progress and it shows the possible outcomes of an epidemic which further helps in forming control strategies [8, 17, 18]. The earliest mathematical model in epidemiology was proposed by Daniel Bernoulli in 1760 on inoculation against smallpox [13]. The 1920s saw the emergence of compartmental models. It can be said that the compartmental models are the basic foundation of mathematical epidemiology. The famous SIR compartmental system was formulated by Kermack and McKendrick [14] in 1927. He studied the Great Plague outbreak during 1665-1666 in London, and the Mumbai outbreak of plague in 1906. In 1906, W.H. Hamer suggested that the spread of infection should depend on the number of susceptible and infectious individuals [11]. He proposed a mass action law for the rate of new infections, and this idea has been basic in compartmental models since that time. It is worth noting that the foundations of the entire approach to epidemiology based on compartmental models were laid, not by mathematicians, but by public health physicians such as Sir R.A. Ross, W.H. Hamer, A.G. McKendrick, and W.O. Kermack between 1900 and 1935. Since then, several researchers have continued this research legacy with their alterations to contribute a significant amount of work [1,2,6,7,9,19,21]. There are various important aspects that affect the spread of disease in the real world. Some of these factors include limited medical resources, re-infection, immunity, isolation, human demographic changes and behavior, co-infection, and so on [3{5,10,12,20]. These aspects make the dynamics of disease spread a com- plex process as they severely affect the emergence, persistence, and eradication of the disease. In this thesis, we study some of these factors and how they complicate the dynamics of disease spread.

Contents

11. JATIN ANAND
Composition Operators and Invariant Subspaces.
Supervisor: Prof. Sachi Srivastava
Th 26103

Abstract

Composition and weighted composition operators have extensively studied on spaces of analytic functions and an enormous amount of literature has been written in this area. The beauty of studying these operators is that they establish a connection between operator theory and theory of analytic functions. Several problems related to analytic functions have been tackled using ideas of operator theory via these operators and vice-versa. One of the most pivotal problems in operator theory which is still unsolved is the "Invariant Subspace problem" for separable Hilbert spaces. The problem is stated as follows: Does every operator on an infinite dimensional separable Hilbert space has a non –trivial invariant subspace? Several approaches have been tried to solve this problem. As quoted by P.R. Halmos [8], "A possible approach is to classify all invariant subspaces of all known operators in the hope of getting an insight that will lead to a proof or a counterexample" In this thesis we a step forward in this direction and characterize common invariant subspaces of composition operators and certain shifts and also extend this in the context of composition operators.

Contents

12. KAPOOR (Shiva)
Stability and Density Aspects in Vector Optimization.
Supervisor: Prof. C.S. Lalitha
Th 26099

Abstract

Vector optimization is a vigorous and expanding branch of applied mathematics which deals with optimization problems comprising of more than one conflicting objective function that are to be optimized simultaneously. Since, the quality of an optimization model depends strongly on the accuracy of the initial data supplied, thus it is important to analyze the behavior of solution sets under small perturbation of basic data. Stability analysis deals with analyzing conditions which result in small changes in the solution sets under small perturbations of the objective function, the constraint set or the ordering cone. The study of evolution of the solution sets under perturbations emerged as a prominent field of study among researchers. The erception of stable problems may vary from one researcher to another. However, usually it is recognized via solution sets having certain properties such as convergence, continuity, essentiality, density and so on. The thesis is dedicated to the extensive study of these varied aspects of stability in vector optimization and comprises of six chapters.

Contents

13. MALHOTRA (Aastha)
Complex Symmetries of Composition and Differential Operators on Analytic Function Spaces.
Supervisor: Dr. Anuradha Gupta
Th 26107

Abstract

The concept of complex symmetric matrices - whose entries are complex numbers which are symmetric about the main diagonal - is frequently encountered in the study of linear algebra and its applications. Not all matrices are complex symmetric in nature but there are some matrices which possess a certain kind of hidden symmetry. There are examples of matrices which are themselves not complex symmetric but are unitarily similar to complex symmetric matrices. The existence of these hidden symmetries is best explained in the framework of complex symmetric operators which is a class of quite well-behaved operators. The complex symmetric operators are characterized by their interactions with certain types of linear operators known as conjugations. A conjugation C acting on a complex Hilbert space H is a map which is conjugate-linear (i.e., C(ax + by) = a C(x) + b C(y) for all x; y 2 H and for all a; b 2 C), involutive (i.e., C2 = I) and isometric (i.e., kCxk = kxk for all x 2 H). A bounded linear operator T on a separable complex Hilbert space H is said to be complex symmetric if there exists a conjugation C such that T = CTC or, we may write, TC = CT. An equivalent de nition for a complex symmetric operator also exists. A bounded linear operator T on H is said to be complex symmetric if there exists an orthonormal basis for H with respect to which the matrix representation of T is a self-transpose (symmetric) matrix. The class of complex symmetric operators is quite diverse. It includes every normal operator, idempotent operator, truncated Toeplitz operator, Hankel operator, Volterra integration operator etc. The theory of complex symmetric operators has ample number of applications within operator theory. Garcia and Putinar [5, 6] gave applications of complex symmetric operators to Jordan canonical models, selfadjoint extensions of symmetric operators, rank-one unitary perturbations of the compressed shift, etc. Their work motivated several researchers within the mathematical community and a number of authors have contributed signi cantly to the study of complex symmetric operators (see [4, 7, 19, 20] and references therein). Chapter one contains the requisite notations and terminologies which have been used in the subsequent chapters. It further gives a brief introduction of complex symmetric operators. In Chapter two, we have de ned a new space by introducing an equivalent norm on the derivative Hardy space. We ave rst proved some basic results related to the properties of this new space. We have further studied some traditional properties (such as boundedness) of the composition operator as well as of the multiplication operator acting on this new space. We have also characterized the weighted composition operators acting on the weighted Hardy space which are complex symmetric with respect to a certain xed conjugation. In Chapter three, we introduce the concept of Toeplitz composition operators on the Hardy space. We have obtained the structure of the symbols of a Toeplitz composition operator so that it is complex symmetric with respect to some xed conjugation de ned on the Hardy space. We have also determined the conditions

under which a complex symmetric Toeplitz composition operator is normal. We have further explored the necessary conditions under which a Toeplitz composition operator becomes hyponormal and normal Chapter four deals with the problem of determining the complex symmetric structure of a weighted composition operator acting on a codimension one subspace of the Hardy space. We have also obtained some conditions under which the concepts of isometry and coisometry are equivalent for a weighted composition operator acting on the given codimension one subspace of the Hardy space. In Chapter five, we have characterized complex symmetric generalized weighted composition operators acting on the Fock space as well as complex symmetric rstorder di erential operators acting on the codimension one subspace of the Hardy space. Both of these operators are unbounded in nature and thus, we have rst proved certain results which are then used to accomplish our target of nding the form of the symbol functions of both of these operators. Moreover, we have obtained the necessary and sufficient conditions under which these unbounded operators are Hermitian.

Contents

14. MOHAMMAD SALMAN
Transitivity, Sensitivity, Devaney's Chaos and Specification Properties in Nonautonomous Dynamical Systems.
Supervisor: Prof. Ruchi Das
Th 26108

Abstract

The theory of dynamical systems is a very well developed and successful mathematical tool to describe time- fluctuating phenomena. It is one of the most applicatory and crucial branches of mathematics which is largely built with the help of analysis, geometry and topology. Its wide application area differs from simple motion of a pendulum to complex climate models in physics and complicated signal processes in biological cells. Over the last few decades, with the revolutionist contributions of Poincare and Lyapunov and with the discovery of chaos and attractors, dynamical systems have gained substantial interest and resulted in considerable developments in this area. One of the most significant constituents of dynamical systems theory is chaos which is closely related to different variants of sensitivity and mixing. The rest paper describing chaos in a mathematically meticulous way was given by Li and Yorke in 1975 [19]. Since then the research on dynamical systems and chaos theory has had a large influence on modern science. A dynamical system mainly contains three essential ingredients, namely, \phase space, time and evolution rule." (i) A phase space is the one in which every possible states, corresponding to a unique point, can be represented. (ii) Time can be discrete or continuous depending on the set of values. (iii) Evolution rule characterizes the development of one state of a system into another over a period of time. If the set of integers (respectively, real numbers) are acting during the evolution rule, then the corresponding system is called as a discrete dynamical system (respectively, continuous dynamical system).

Contents

15. SAINI (Manisha)
Growth of Solutions of Complex Differential Equation with Entire Coefficients.
Supervisor: Prof. Sanjay Kumar
Th 26097

Abstract

The present thesis deals with complex differential equations with entire functions as coefficients. The principal aim of this work is to investigate the growth of solutions of these equations. We impose conditions on the coefficients of differential equations so that their each solution is of infinite order of growth. Moreover, we investigate the hyper-order of growth of these solutions. The substantial work of this thesis deals with the homogeneous linear differential equations. In addition, we have also proved some results for non-homogeneous linear differential equations. The thesis is divided into four chapters followed by appendices at the end. We have presented our work in Chapters 2 to 4. The chapter wise arrangement is as follows: Chapter 1 provides an overview of the concept of differential equation from its historical point of view. We discuss several known results in this area that we shall frequently use in the rest of the thesis. Chapter 2 comprises of the study of second order linear differential equations wherein the coefficient of f ′ is a transcendental entire function and the coefficient of f is taken to be a polynomial. We further assume that these two coefficients are transcendental entire functions and have different order of growth. Also, the hyper-order of these solutions are obtained. We have extended our result from second order linear differential equations to higher order linear differential equations. 3 4 In Chapter 3, we continue our study to find the properties satisfied by the coefficients of second order linear differential equations so that solutions of these equations possess infinite order of growth and finite hyper-order of growth. These results generalize our result discussed in the previous chapter where we have assumed the coefficients with different order of growth. We have proved that if the order of growth of these coefficients are equal then also the conclusion holds. These results have been extended from second order linear differential equations to higher order linear differential equations. Chapter 4 consists of the results wherein the coefficient of these differential equations has a finite Borel exceptional value. We investigate the second order non-homogeneous linear differential equations so that all the solutions of these equations have infinite order and finite hyper-order of growth. The results for higher order linear differential equations are an extension of our results for second order linear differential equations.

Contents

16. SURBHI

Local Versions of Operator Systems and Ternary Rings of Operators.

Supervisors: Prof. Ajay Kumar and Prof. Mukund Madhav Mishra

Th 26101

Abstract

The theory of operators has been one of the crucial topics in functional analysis. In the last few decades, this theory showed a rapid growth and yet to reach a saturation point. Operator theory plays a very important role in quantum information theory as well. With the initiation of J. von Neumann and F. J. Murray the last century has witnessed a chase among functional analysts to find non-commutative counterparts of the classical spaces. Initially, they were only curious about the non-commutative analogues of the function spaces $L\infty(K, \mu)$ and C(K), consisting of respectively complex valued bounded measurable and continuous functions on a compact Hausdorff space K, which correspond to von Neumann algebras and C∗-algebras. These are ∗-subalgebras of the space B(H) consisting of all bounded linear maps on a Hilbert space H. In the recent years, subspaces and self-adjoint subspaces of B(H) attracted many mathematicians globally. It was observed that the crucial properties that a subspace and a self-adjoint unital subspace inherit from B(H) are the matrix norms and the matrix order of the parent space respectively. In fact, Z. J. Ruan observed that a vector space equipped with matrix norms satisfying certain conditions (Ruan's axioms) may be realized as a subspace of B(H), where H is a Hilbert space. Further M. D. Choi and E. G. Effros stated that a ∗-linear space together with a considerable matrix order can be embedded inside B(H) as a ∗-closed subspace containing the unit. The morphisms in these categories are respectively called completely bounded and completely positive maps; in simple words we can say these are linear maps which respect the matrix norms and the matrix ordering respectively. One can observe that already known classical Banach space and C∗-algebraic techniques have helped extensively in exploring these two categories.

Contents

17. TAMVI

Stability Analysis and Optimal Intervention for HIV-TB CO-Infection Including Fractional Order and Delay in Treatment.

Supervisor: Prof. Rajiv Aggarwal

Th 26100

Abstract

The main objective of mathematical modeling of infectious diseases is to identify and study the factors that influence the spread of the disease and to predict the future dynamics of a particular disease or combination of diseases under consideration. Mathematical modeling of infectious diseases is significantly used in formulating and evaluating strategies to control and prevent their spread in the susceptible population. Throughout the world, more so in the developing world, there are number of deadly infectious diseases which are severely affecting the lifespan of the human population. Acquired Immuno Deficiency Syndrome (AIDS) is one of such

deadly diseases caused by the Human Immunodeficiency Virus (HIV). Tuberculosis (TB) is an infectious disease and in humans it is mainly caused by Mycrobacterium tuberculosis. HIV and TB intensify the progression of each other (Sharomi et al. (2008)). In comparison to people living without HIV, the chances of occurrence of active TB in HIV positive individuals is 15-22 times more. The main content of this thesis is based on different mathematical models on HIVTB co-infection by incorporating treatment and screening of unaware infectives to see the impact of HIV and TB on each other. Optimal control measures to optimize the number of infectives and to propose the ideal timings of treatment for both the disease have also been introduced. The thesis is divided into six chapters followed by references at the end. The chapter wise organization of the thesis is as follows:

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