

# CHAPTER 30

## MATHEMATICS

### Doctoral Theses

01. AGGARWAL (Rachna)  
**Isometry Groups of Infinite Dimensional Complex Hyperbolic Spaces.**  
Supervisors: Dr. Ratikanta Panda and Dr. Mukund Madhav Mishra  
Th 26762

#### *Abstract*

We study the group of holomorphic automorphisms of the open unit ball, of a complex Hilbert space, equipped with the Caratheodory metric. In specific, we examine the conjugacy classes and centralizers in the group, based on the fixed point classification of isometries in the group. Then we generalize the above model to open unit ball, of  $B(K,H)$ , equipped with the Caratheodory metric. Here  $B(K,H)$  is the Banach space of bounded linear operators from a complex Hilbert space  $K$  to another complex Hilbert space  $H$ . Here we characterize the linear normal isometries in terms of its fixed points. We mainly carry out our studies by assuming  $K=C^n$ .

#### *Contents*

1. Introduction 2. Some special sub-classes of the group of isometries. 3. Conjugacy classes in the group of isometries 4. Centralizers of elements in the group of isometries 5. Holomorphic automorphisms of the unit ball of  $B(K,H)$  107. Bibliography. Index.

02. ALKA  
**Various Aspects of Point –Set Domination in Graphs.**  
Supervisor: Dr. Ranjana Jain  
Th 26763

#### *Abstract*

Graph theory is a mathematical structure that studies interactions, conflicts and relationships between discrete objects. The theory of domination is one of the most significant and historically rich topics of graph theory. There are more than 100 variants of domination and domination-related parameters that are being studied extensively. This thesis demonstrates that there is an enormous and far-reaching scope of research in newer directions of the domination theory. The thesis mainly deals with a variant of domination, called point-set domination. We obtain some upper and lower bounds, on point-set domination number of a graph in terms of its order, maximum degree, girth and diameter. Further, we completely characterize extremal for the lower bound and present some classes of extremal graphs for the upper bound. The thesis further adds to the literature towards the problem of determining graphs which possess an independent psd-set. Further, a few results on line-set domination given by Rao and Laxmi in 1999 are improved. Since the notion of line-set domination is equivalent to the notion of point-set domination in line graphs, the thesis extends the work of Rao and Laxmi on line-set domination and also answers the problem of characterizing graphs whose line graphs possess an

independent point-set dominating sets. The thesis can also be seen as a seed for a variant of point-set domination, called secure point-set domination. Motivated from the definition of secure domination, the thesis introduces a new protection strategy in graphs termed as secure point-set domination. We characterize graphs with secure point-set domination number 2. Further, we obtain a forbidden subgraph characterization for graphs with secure point-set domination number  $n - 2$ .

*Contents*

1. Introduction 2. Point-set domination in graphs 3. Independent point-set domination in graphs 4. Independent point-set domination in line graphs 5. Secure point-set domination in graphs 6. Conclusion and further scope. Bibliography

03. AMIT KUMAR

**Dynamical Study and Exact Solutions for the Higher-Dimensional nonlinear Evolution Equations Using Various Analytical Techniques.**

Supervisor: Dr. Sachin Kumar

Th 26764

*Abstract*

In this thesis, we study the applications of various analytical techniques to the nonlinear evolution equations, which describe some of the important nonlinear physical phenomena. Nonlinear evolution equations are a particular case of nonlinear partial differential equations representing the time-dependent behavior of systems whose evolution is influenced by nonlinear interactions between different variables. These equations involve nonlinear terms that make them more complicated than linear evolution equations, which can be solved using straightforward techniques. Partial differential equations (PDEs) can be regarded as evolution equations on an infinite dimensional state space. So, our primary objective in this thesis is to apply a few different analytical techniques, namely the generalized exponential rational function method, the generalized Kudryashov method, the generalized Riccati equation mapping method, and the Lie groups of transformations method to obtain a variety of exact solutions of some nonlinear partial differential equations. Furthermore, it discusses the dynamic investigations of obtained solutions for the equations under consideration. The investigations carried out in this thesis are confined to the applications of the used method for five nonlinear partial differential equations, which are the (1+1)-dimension strain wave equation, the (1+1)-dimensional cold bosonic atoms in a zig-zag optical lattice model, the (2+1)-dimensional modified Veronese web equation, the (3+1)-dimensional generalized KP equation, and the (4+1)- dimensional Fokas equation.

*Contents*

1. Literature review and Introduction 2. Exact solitary waves solutions of the (1+1)-dimensional strain wave equation 3. Exact optical soliton solutions of the (1+1) dimensional of the zig-zag optical lattice mode 4. Exact closed-form solutions of (2+1)- dimensional modified Veronese web equation 5. Exact solutions of the (3+1)-dimensional generalized KP equation 6. Abundant exact solutions of the (4+1)-dimensional fokas equation. References.

04. BARKHA

**Compact-  $G_n$ - Open, -Point and Point-Open Topologies.**

Supervisor: Prof. A.R. Prasanna

Th 26766

*Abstract*

Let  $C(X, \mathbb{R})$  denotes the family of all continuous functions from a space  $X$  to the space  $\mathbb{R}$ , where  $\mathbb{R}$  denotes the set all real numbers with its usual topology. The idea of defining a topology on the set  $C(X, \mathbb{R})$  came from the convergence of sequence of functions in calculus. There are various topologies defined on  $C(X, \mathbb{R})$ . In [2], R. Arens and J. Dugundji introduced a topology on  $C(X, \mathbb{R})$  called set-open topology. The subbasic open set of this topology is  $[K, U] = \{f \in C(X, \mathbb{R}) : f(K) \subset U\}$ , where  $K$  is a non empty open subset in the space  $X$  and  $U$  is open in  $\mathbb{R}$ . There are many set-open topologies but point-open topology and compact-open topology are extensively studied and used in various fields. The point-open topology, denoted by  $p$ , is also called topology of pointwise convergence and this topology is as old as calculus (see [3] and [17]). The compact-open topology, denoted by  $k$ , was introduced by Fox [5]. The space  $C(X, \mathbb{R})$  equipped with  $p$ -topology is denoted by  $C_p(X, \mathbb{R})$  and the space  $C(X, \mathbb{R})$  equipped with  $k$ -topology is denoted by  $C_k(X, \mathbb{R})$ . Many studies are done also on the topologies that are finer than both  $p$  and  $k$  topologies such as  $C$ -compact open topology, pseudocompact open topology and  $\sigma$ -compact open topology (see [13], [15] and [19]). It is well known that  $p$ -topology and  $k$ -topology coincides on  $C(X, \mathbb{R})$  if and only if all the compact subsets in the space  $X$  are finite. It implies that there is a considerable gap between the  $p$  and  $k$  topologies and the condition on the space  $X$  is quite extreme in nature. So it is natural to find out and study about the topologies that lies between  $p$  and  $k$  topologies. S. Kundu and R.A. McCoy in [14] defined this type of topologies called Weak and support-open topologies on  $C(X, \mathbb{R})$  and further these topologies are studied in [12]. But again the natural question arises that whether there exists a topology that is coarser than  $k$  topology but may or may not be finer than  $p$  topology from view point of topology. The compact- $G\delta$ -open topology ([6] and [8]) is such type of topology. It is always coarser than  $k$  topology and it need not be finer than  $p$  topology on  $C(X, \mathbb{R})$  for a completely regular space  $X$ . P. Garg and S. Kundu studied compact- $G\delta$ -open topology  $kz$  on  $C(X, \mathbb{R})$  and we analogously define  $kz$  topology on  $C(X, Y)$  by taking open sets from the space  $Y$ , where  $C(X, Y)$  denotes the family of continuous functions from the space  $X$  to the space  $Y$ . One of our main aim is to study  $kz$  topology on  $C(X, Y)$ , where  $Y$  is a locally convex space [25]. The space  $C(X, Y)$  equipped with  $kz$  topology is denoted by  $C_{kz}(X, Y)$ . Considering an arbitrary locally convex space  $Y$  can lead to many general results on  $C_{kz}(X, Y)$ .

*Contents*

1. Introduction 2. Compact  $G\delta$ - open topology on  $C(X, Y)$  3. Open- point, point-open and bi-point-open topologies of  $C(X, Y)$  and  $C(X, X)$  4. Bi-compact- $G\delta$ - open topology on  $C(X, \mathbb{R})$ . References.

05. CHAUDHARY (Bipin Kumar)  
**Discontinuous Waves in Quasi-Linear Hyperbolic Systems and Riemann Problem.**  
 Supervisor: Dr. Randheer Singh  
Th 26767

*Abstract*

In the thesis, we have considered the quasilinear hyperbolic systems of one-dimensional planar and radially symmetric flow of van der Waals stiffened relaxing gases accompanying the mixture of crystalline solids, two-phase Chaplygin flows, spherically symmetric self-gravitating interstellar van der Waals gas clouds accompanying with dust particles and real isothermal gases with dust particles. The thesis consists of six chapters in which the first chapter is introductory and the

remaining five chapters constitute the main part of the thesis which are briefly described as follows. The second chapter investigated the collapse of converging shocks for the radially symmetric flow of van der Waals stiffened relaxing gases by using the Lie group invariance method. The distribution of flow variables in the medium just behind the shock for the cases of power-law and exponential law shock paths are analyzed. The effects of parameters involved in the system, on the self-similar exponent and on the converging, shocks are also carried out. In the third chapter, the intrinsic phenomena of concentration and cavitation are investigated in the Riemann problem to the pressureless isentropic Chaplygin two-phase flows by using the three-parameter flux approximation. The fourth chapter investigated the interaction problem of the steepened wave with a strong shock in the planar and radially symmetric flow of van der Waals stiffened relaxing gases. An exact solution to the considered system is used to investigate the evolution of steepened wave. The amplitudes of the reflected and transmitted wave along with the bounce in shock acceleration, originating from a collision between steepened wave and strong shock are also computed. In the fifth chapter, we deal with the one-dimensional steepened wave in the inviscid, self-gravitating and spherically symmetric flow of interstellar van der Waals dusty gas clouds via Guderley's exact similarity solution. A transport equation for the jump in velocity gradient is deduced. The effects of van der Waals excluded volume and dusty gas parameters, present in the system under consideration, on the steepened wave are determined in detail. In the last chapter, the solution to the Riemann problem for isothermal flow of real gases with dust particles involving elementary waves is analyzed. The expressions for elementary waves in terms of flow variables are determined. Further, the properties of rarefaction wave and shock waves along with all possible solution patterns to Riemann problem are investigated.

#### *Contents*

1. Introduction 2. Converging shocks in van der waals stiffened relaxing gases 3. Concentration and cavitation of Riemann solutions to two-phase chaplygin flows under vanishing pressure and flux approximation 4. Interaction of steepened wave with a strong shock in van der waals stiffened relaxing gases 5. Spherical steepened wave in interstellar van der waals dusty gas clouds 6. The Riemann problem for real isothermal gases with dust particles. Bibliography.

06. CHAUHAN (Dimpy)  
**Some Properties of Quasigroups and Applications in Designing Cryptographic Primitives.**  
 Supervisors: Dr. Rashmi Verma and Indivar Gupta  
Th 26768

#### *Abstract*

Cryptography [16, 32, 41] comprises the study of mathematical techniques related to aspects of information security such as confidentiality, data integrity, entity and data-origin authentication. Designers employ cryptographic primitives as the fundamental building blocks for constructing cryptosystems. Hence, cryptographic primitives are designed to perform a single extremely specific task in a well-defined and highly reliable manner. One of the current research areas in cryptography is to design cryptographic algorithms using a novel approach. Over the past two decades, the applications of quasigroups in cryptography have evolved at a high speed and have received a lot of attention of the researchers in this area. Quasigroups are widely used in designing various cryptographic primitives, namely S-boxes [34, 45], block cipher [3, 4, 27, 42, 46], stream cipher [28, 36, 43], hash functions [15, 31],

secret sharing schemes [10], message authentication codes [33], and many more [35]. The structure, properties (viz. non-associativity and non-commutativity) and the existence of a large number of quasigroups enable them to be applied in various fields such as cryptography, coding theory, telecommunications, experimental designs, etc. Quasigroups' closure and inversion properties make them useful tools for constructing an efficient cryptosystem. Also, the lack of associativity facilitates the creation of one-way functions. Furthermore, because of the lack of commutative and associative properties in a quasigroup, many well-known traditional cryptographic approaches to attack quasigroup based cryptographic designs can not be applied directly. The quasigroups have an equivalent algebraic and combinatoric structure to Latin squares. The theory of Latin squares is well studied and has several fascinating areas of research, viz. mutually orthogonal Latin squares, transversals of Latin squares and Latin subsquares [6, 18, 24]. Thus, quasigroups and Latin squares play a significant role in designing cryptographic primitives. In 1940, Garrison [12] introduced the concept of quasigroup nuclei. The nuclei of a quasigroup comprises of left nucleus, right nucleus and middle nucleus of a quasigroup. A left/ right/ middle nucleus of a quasigroup measures how far a quasigroup is from a group by measuring the near-associativity of the quasigroup. The quasigroup nuclei have been generalized to the A-nuclei of a quasigroup and their properties have been extensively studied by Kepka [23], Keedwell, Anthony [22], Shcherbacov [39], etc. For  $\sigma \in S_3$ , the left, right and middle  $\sigma$ -A-nuclei of a quasigroup have been defined in [40] which is not a generalization of its left, right and middle A-nuclei, respectively. To the best of our knowledge, the properties of  $\sigma$ -A-nuclei of a quasigroup have never been explored by researchers. In this thesis, we redefine left, right and middle  $\sigma$ -A-nuclei of a quasigroup as a generalization of its left, right and middle A-nuclei, respectively. Several properties of quasigroups including the properties of A-nuclei and  $\sigma$ -A-nuclei of a quasigroup have been investigated.

#### *Contents*

1. Introduction 2. A- nuclei of a quasigroup and their properties 3.  $\partial$ - nuclei of a quasigroup and their properties 4. Relations between.  $\partial$ - nuclei and A-nuclei 5. Construction of cryptographically strong S-boxes from ternary quasigroups of order 4 6. Cryptographic primitives based on quasigroups. Conclusion and future research directions. Bibliography.

07. GAURAV KUMAR

#### **Variants of Rothberger, Hurewicz and Menger Topological Spaces.**

Supervisors: Prof. Brij Kishore Tyagi and Dr. Sachin Vashistha and

Th 27225

#### *Abstract*

The thesis deals with the study of several variant of Rothberger, Hurewicz and Menger type covering properties. It consist of five chapters. 1. Introduction 1.1 Historical Development and Motivation 1.2 Preliminaries and basic Definitions 1.3 Summary of Thesis 2. Variants of Rothberger Spaces 2.1  $\theta$ -Rothberger Spaces and  $\alpha$ -Rothberger Spaces 2.2 Almost  $\alpha$ -Rothberger Spaces 2.3 Strongly Star  $\alpha$ -Rothberger and Strongly Star  $\theta$ -Rothberger Spaces 2.4 Preservation under Subspaces and Mappings 3. Variants of Hurewicz Spaces 3.1  $\theta$ -Hurewicz Spaces and  $\alpha$ -Hurewicz Spaces 3.2 Characterizations of Variants of Hurewicz Spaces 3.3 Preservation under Subspaces and Mappings 4. Variants of Menger Spaces 4.1 Semi-Menger and Almost Semi-Menger Spaces 4.2  $\theta$ -Menger Spaces 4.3  $\theta$ -Almost Menger and  $\theta$ -Weakly Menger Spaces 5. Weakly Strongly Star-Menger Spaces 5.1 Weakly Strongly Star-Menger Spaces 5.2 Preservation under Subspaces and Product Spaces 5.3 Preservation under Mappings

*Contents*

1. Introduction 2. Variants of Rothberger spaces 3. Variants of Hurewicz spaces 4. Variants of Menger spaces 5. Weakly strongly star-Menger spaces. Reference.

08. GUPTA (Bhawna Bansal)  
**Slant Hankel Operators on the Lebesgue Space of  $n$ -Torus.**  
 Supervisor: Dr. Gopal Datt  
Th 26769

*Abstract*

We have studied slant Hankel operators in the multidimensional setup, that is, on  $L^2(\mathbb{T}^n)$ , the Lebesgue space of  $n$ -torus and their compression on  $H^2(\mathbb{T}^n)$ , the Hardy space of  $n$ -torus. Here  $\mathbb{T}^n$  represents the Cartesian product of the  $n$ -copies of the unit circle  $\mathbb{T}$ . A detailed study of various properties of these operators along with their generalized versions is dealt within the thesis. The scope of the thesis also covers a discussion on Hankel operators on  $H^2(\mathbb{T}^n)$ . Commutativity of Hankel and Toeplitz operators in  $n$ -variables set-up is explored. Various examples are presented to show that certain results concerning commutativity between these operators which hold good in the one-variable case may not hold in the multivariable case.

*Contents*

1. Introduction 2. Hankel operators on the Hardy space of  $n$ -torus 3. Slant Hankel operator on  $L^2 \mathbb{T}^n$  4. Slantification of Hankel and compression of slant Hankel operators 5. Generalizations of slant Hankel operators 6. Slant Hankel operators of multivariate order. Appendix: further scope of research. Reference.

09. GUPTA (Meenakshi)  
**Well-Posedness and Stability in Set Optimization.**  
 Supervisor: Prof. Manjari Srivastava  
Th 26770

*Abstract*

Well-posedness of an optimization problem means that the variables whose objective function values are close to the optimal value ultimately lie close to solution of the problem. This is a well researched area in optimization theory. It plays an important role in the study of stability of optimization problems which helps to analyze the behaviour of its solution sets when the initial data of the problem is perturbed. In literature, the stability aspects of vector optimization problems have been studied by taking into account the perturbations of the ordering cone, the feasible set and the objective function. The thesis is devoted to the study of well-posedness and some aspects of stability in set optimization, in particular, convergence of minimal solution sets. Throughout the thesis, we assume that  $(X, \|\cdot\|_X)$  and  $(Y, \|\cdot\|_Y)$  are real normed spaces and  $K$  is a closed convex pointed cone in  $Y$  with nonempty interior unless specified otherwise. We consider the following set optimization problem

$$\begin{aligned} & \text{(SP) } \min_K F(x) \\ & \text{subject to } x \in M \end{aligned}$$

where  $F : X \Rightarrow Y$  is a set-valued mapping and  $M$  is a nonempty set in  $X$ . The thesis comprises of five chapters. The first chapter is introductory and presents some basic notions and concepts which are required in the subsequent chapters. We also provide a brief summary of certain well-posedness and stability aspects in scalar,

vector and set optimization. Chapter 2 entitled Well-Posedness through Scalarization is based on the paper by Gupta and Srivastava [3]. It deals with the study of two types of pointwise well-posedness and one type of global well-posedness for the set optimization problem (SP) based on the upper set order relation. These well-posedness are applicable for solid as well as for nonsolid optimization problems. In this chapter, we assume that  $(X, d)$  is a metric space,  $(Y, \|\cdot\|_Y)$  is a real normed linear space and  $K$  is a closed convex pointed cone in  $Y$  which may have an empty interior. This chapter comprises of two sections. In Section 2.1, three kinds of well-posedness for the problem (SP) have been defined and a few necessary and sufficient conditions for these well-posedness are established. Chapter 3 entitled Levitin–Polyak Well-Posedness and Stability is based on the paper by Gupta and Srivastava [5]. This chapter, in three sections, is devoted to the study of Levitin–Polyak well-posedness and stability aspects for the set optimization problem (SP) with respect to the upper set order relation. The Levitin–Polyak well-posedness introduced in this chapter generalizes the notion of global well-posedness defined in the previous chapter by allowing the minimizing sequences to stay outside the feasible set. The stability results have been investigated by considering the perturbations of the ordering cone and the feasible set of the problem (SP).

#### *Contents*

1. Introduction 2. Well-posedness through scalarization 3. Levitin- polyak well-posedness and stability 4. Approximate solutions and levitin – polyak well-posedness 5. Hadamard well-posedness and stability. Bibliography.

10. GUPTA (Naveen)  
**Squeezing Function and its Generalizations.**  
 Supervisor: Dr. Sanjay Kumar  
Th 26771

#### *Abstract*

Riemann mapping theorem says that every proper simply connected domain in complex plane is biholomorphic to the unit disc. This result does not hold in  $C_n$ ,  $n > 1$ . Poincaré has shown that the unit ball is not biholomorphic to the unit polydisk. This is one of the leading reasons that the study of biholomorphic invariants becomes important in  $C_n$ ,  $n > 1$  and squeezing function is one of such biholomorphic invariants. It has been a topic of great interest to many researchers since its inception in the work of Yau et al [27], [28], Yeung [43] in the context of differential geometry and several complex variables respectively. For a bounded domain  $D \subseteq C_n$ , squeezing function [7]  $SD$  is defined as  $SD(z) := \sup\{r > 0 : B_n(0, r) \subseteq f(D) : f \in O(D, B_n), f(z) = 0\}$ , where  $B_n(0, r)$  denotes ball of radius  $r$  centered at the origin, the unit ball is denoted by  $B_n$  and  $O(D, B_n)$  is the set of injective holomorphic mappings  $f : D \rightarrow B_n$ . The squeezing function is biholomorphic invariant is the straightforward consequence of the definition. It is known that the unit ball and the unit polydisk are not biholomorphic to each other, so it seems natural to consider the squeezing function when the embeddings are taken into the unit polydisk instead of the unit ball. In [15], we took the investigation in this setup. Rong and Yang, in [38] presented generalized squeezing function, where instead of unit ball/unit polydisk, the authors considered balanced domains. In [14], we considered more general object, naming them  $d$ -balanced squeezing function involving  $d$ -balanced domains. For a bounded domain  $D \subseteq C_n$ , B. L. Fridman in [12], [13] introduced another similar biholomorphic invariant, now known as Fridman invariant. It is denoted by  $hD(z, B_n)$  (this is the notation used in Fridman's article) and is defined as follows:

$h_D(z, B_n) := \inf\{1/r : B_{cD}(z, r) \subseteq f(B_n, D), f \in O(B_n, D)\}$ , where  $c_D$  is the Carathéodory pseudodistance and  $B_{cD}(a, r)$  is the Carathéodory ball centered at  $a$  of radius  $r > 0$ . It should be noted here that in the above definition in place of the unit ball any bounded homogeneous domain can be considered. In this thesis, we study these invariants, their properties and relation between them. The thesis is divided into five chapters out of which the latter four chapters deal with the research work carried out by us. The chapterwise arrangement is as follows: Chapter 1 begins with the introduction of squeezing function, Fridman invariant and their properties. We have tried to arrange them in proper historical context as available in the literature. Chapter 2 entitled “Squeezing function corresponding to polydisk” discusses the case where the embeddings are taken into the unit polydisk instead of the unit ball. Some properties, a few examples and its relation with the squeezing function (in the context of the unit ball) is also discussed. In the Chapter 3 entitled “d-balanced squeezing function”, we begin with the discussion on ‘generalized squeezing function’ and present a more general set up related to d-balanced domains. We discuss the concept of the d-balanced squeezing function and 2 present some properties related to it. We also study its relation with the corresponding Fridman invariant. Chapter 4 which is entitled “Relation between  $S_D, S_D^\Omega, S_{d,D}^\Omega$ ” clarifies how these generalizations stand in relation with the classical squeezing function. In particular, results presented in this chapter implies that the the generalizations agree with the concept of holomorphic homogeneous regular domain in the sense that there is no ambiguity when one shifts from one set up to the other. Chapter 5 entitled “Examples of nonplurisubharmonic d-balanced squeezing function” begins with the construction of a domain, which has nonplurisubharmonic d-balanced squeezing function. We then present a class of domains having plurisubharmonic squeezing function (or more generally, d-balanced squeezing function).

### Contents

1. Introduction 2.Squeezing function corresponding to polydisk 3. d –balanced squeezing function 4. Relation between  $S_D, S_D^\Omega, S_{d,D}^\Omega$  5. Examples of nonplurisubharmonic d-balanced squeezing function. References.

11. JINDAL (Divya)  
**Frames with Several Generators.**  
 Supervisor: Dr. Lalit Kumar  
Th 27222

### Abstract

The study of frames from the Weyl-Heisenberg group and the extended affine group. We give necessary and sufficient conditions for the existence of frames from the Weyl-Heisenberg group and the extended affine group and give the interplay between these frames. Varied perturbation results for frames from the Weyl-Heisenberg group and the extended affine group are also given. Sufficient conditions for the sums of the frames and their applications to the frame algorithm are provided. Necessary and sufficient conditions for the existence of nonstationary frames of translates and its applications in terms of the construction of the frames from the Weyl-Heisenberg group and the extended affine group are given. Sufficient conditions for nonstationary Riesz bases of the space  $L^2(\mathbb{R})$  are also given. Nonstationary frames of matrix-valued Gabor systems and wavelet systems in the matrix-valued function space  $L^2(\mathbb{R}, C^{(s \times s)})$  are discussed. Matrix-valued frame preserving maps in terms of adjointability, with respect to the matrix-valued inner product, of bounded linear operators acting on  $L^2(\mathbb{R}, C^{(s \times s)})$  is provided. Specifically, we investigate bounded linear operators acting on  $L^2(\mathbb{R}, C^{(s \times s)})$  and their impact on the frame properties

on nonstationary matrix-valued Gabor systems. Furthermore, we show that the image of a matrix-valued Gabor frame under a bounded, linear, and invertible operator on  $L^2(\mathbb{R}, C^{s \times s})$  may not constitute a frame for  $L^2(\mathbb{R}, C^{s \times s})$ . In this direction, we provide sufficient conditions for bounded linear operators on  $L^2(\mathbb{R}, C^{s \times s})$  that can preserve frame conditions.

### Contents

1. Introduction 2. Frames from the weyl- heisenberg group and the extended affine group 3. Linear combinations of frames from the Wey- Heisenberg group 4. Nonstationary frames of translates 5. Matrix-valued nonstationary frames from the weyl-heisenberg group and the extended affine group. Reference.

12. KANSAL (Arpit)

#### **C\*- Ternary Rings and their Tensor Products.**

Supervisors: Prof. Ajay Kumar and Dr. Vandana

Th 26772

### Abstract

A (concrete)  $C^*$ -algebra is a norm-closed self-adjoint subalgebra of  $B(H)$ , the space of bounded linear operators on a complex Hilbert space  $H$ . Since last few decades, there has been great interest in considering subspaces of  $B(H,K)$ , the bounded linear operators from Hilbert space  $H$  to Hilbert space  $K$ , which are closed under a triple product of its elements, e.g. (i)  $(a, b, c) \rightarrow ab^*c$ , (ii)  $a \rightarrow aa^*a$ . In the literature these spaces have been called ternary algebras [20] or ternary rings of operators (TROs) [26] and  $J^*$ -algebras [19]. Hestenes proved that in the finite dimensional setting TROs can be faithfully represented as direct sums of spaces  $M_{m,n}$  of  $m \times n$  complex matrices. At present a great deal is known about the general theory of TROs and their applications, see Harris [19], Hamana [17] and Effros-Ozawa-Ruan [16]. This class includes  $C^*$ -algebras. TROs carry a natural operator space structure. An operator space is a closed subspace  $V$  of  $B(H)$ , for some Hilbert space  $H$ . It follows from the definition that  $V$  inherits a matricial structure  $B(H)$  by identifying  $M_n(V)$  as a subspace of  $M_n(B(H)) = B(H^n)$ . One may refer to [3, 6, 7, 11, 13, 15] for basic development of operator spaces. The first systematic study on structure of ternary subalgebras of  $B(H,K)$  which are closed in the norm topology or in the weak operator topology was begun with the work of Zettl [39]. A  $C^*$ -ternary ring ( $C^*$ -tring)  $(M, [., ., .], \|\cdot\|)$  consists of a complex Banach space  $(M, \|\cdot\|)$  and a ternary product  $[., ., .] : M^3 \rightarrow M$  which is linear in the first and third variable, conjugate linear in the second variable and associative in the sense that  $[x, y, z], u, v = [x, y, [z, u, v]] = [x, [u, z, y], v]$  Moreover the norm satisfies  $\|[x, x, x]\| = \|x\|^3$ ,  $\|[x, y, z]\| \leq \|x\|\|y\|\|z\|$ .  $C^*$ -trings were first introduced by Zettl[39], and have been intensively studied in [1, 28, 30, 35]. Clearly, every TRO and in particular  $B(H,K), M_{n,m}$  and any  $C^*$ -algebra is a  $C^*$ -tring. An anti-TRO is also a  $C^*$ -tring defined same as TRO, except that the multiplication operation is  $[x, y, z] := -xy^*z$ . Zettl [39] proved that every  $C^*$ -tring is the directsum of two  $C^*$ -subtrings  $M^+$  and  $M^-$ , in such a way that  $M^+$ (respectively,  $M^-$ ) is isomorphic (respectively, anti-isomorphic) to a TRO. Moreover, this decomposition of  $M$  is unique. Embedd  $M^+$  in  $B(H^+)$  and  $M^-$  in  $B(H^-)$ , whence  $M \subseteq B(H^+ \oplus H^-)$ , this implies that every  $C^*$ -tring attains a natural operator space structure. Therefore, the theory of operator spaces gives us a new way to look at  $C^*$ -trings. It is known that  $C^*$ -trings share many properties of  $C^*$ -algebras. For example, it is known (see [1]) that every ternary homomorphism must be a contraction. Moreover, every injective ternary homomorphism must be an isometry. Recent work on the topic can be found in [1, 4, 17, 18, 19, 21, 24, 26, 28, 30, 33, 35, 38, 39].

*Contents*

1. Introduction 2. Ternary rings of operators and their inductive limits 3. C\*-ternary rings and their associated C\*-algebras 4. Ideals of C\*-ternary rings 5. Tensor product of C\*-ternary rings. Bibliography. Index.

## 13. KASHIF AHMED

**Some Aspects of Tropical Algebra.**

Supervisors: Dr. Saibal K. Pal and Dr. Radha Mohan

Th 26773

*Abstract*

Tropical Mathematics, a relatively new entrant in the field of algebra, can be broadly divided into two areas - Tropical Algebra, an analogue of linear algebra that is mostly concerned with matrices over tropical semirings, and Tropical Geometry which deals with the geometry of spaces over tropical semirings. This thesis pertains to the former branch and its implementation in key exchange schemes. This contemporary branch of algebra has recently generated a lot of interest among researchers in areas concerned with cryptography for the computational efficiency it provides while maintaining the security levels of classical cryptographic primitives. We begin with a brief discussion on tropical semirings along with some of the properties of tropical matrices and its determinants. This is followed by an introduction of some newly discovered semiring structures with operations ranging from the number of prime factors of two whole numbers, the sum of divisors of two whole numbers, to defining tropical operations over the cartesian product of sets. We then discuss some intriguing properties of these semirings and solve systems of linear equations (both homogeneous and non-homogeneous) over one of the newly discovered semirings. The section concludes with theorems on the commutativity of matrices over the same semiring structure. Thereafter, we analyse multiple tropical key exchange schemes proposed in the last decade along with various successful algebraic attacks over them. This also includes original ideas proposed by us on cryptanalyses of seven such protocols, clearly suggesting that tropical platforms in their present form are not suited for cryptographic protocols and either require some modification in the existing operations or should be combined and used with classical algebraic structures. The last chapter of the thesis consists of a couple of key exchange schemes proposed by us that are based on the new semiring structures. The security analysis of these primitives suggests that they offer better alternative to tropical key exchange schemes, both against brute-force and linear algebra attacks. We conclude the thesis by stating some major highlights of our research along with several recommendations for future work.

*Contents*

1. Introduction 2. Some new semiring structures 3. Tropical key exchange schemes and their cryptanalyses 4. Some new key exchange schemes. Bibliography. Appendix.

## 14. KAUR (Ramanpreet)

**On Certain Aspects of Fatou Components and Escaping Set of a Transcendental Entire Function.**

Supervisor: Prof. Sanjay Kumar

Th 26774

*Abstract*

The study of dynamics of a transcendental entire function began with the work of Fatou in 1920s, who observed that certain dynamical properties which hold for rational functions may not get lifted to the setting of transcendental entire functions. He asked whether the Fatou sets coincide for a pair of permutable transcendental entire functions. Apart from Fatou, majority of the initial work in transcendental dynamics was done by Baker. For polynomials, infinity is always a super-attracting fixed point. This, in particular, gives us existence of an unbounded Fatou component. Baker posed that if a transcendental entire function has order at most  $\frac{1}{2}$  and is of minimal type, then it has no unbounded Fatou components. The other researchers who contributed remarkably to the area include Bergweiler, Eremenko, Fagella, Lyubich, Rippon, Rempe, Sixsmith, Stallard, and many others. Eremenko formally studied an escaping set, and proved certain properties of the same. There has been a lot of work done in dynamics of several variables by Abate, Bedford, Fornæss, Peters, Raissy, Sibony, Smillie, and many others. The main aim of this thesis is to investigate Baker's question on non-existence of unbounded Fatou components, escaping set of composite transcendental entire function, and to provide examples of Baker domains and wandering domains in several variables. The first chapter is introductory and includes almost all necessary machinery with suitable references. We first give a short historical background of the subject followed by preliminary results on Fatou-Julia theory. The second chapter deals with the question raised by Baker about the functions with no unbounded Fatou components. We provide a partial answer to this question of Baker. The proof requires the construction of a couple of sequences of real numbers with certain properties. To obtain such sequences, Wiman-Valiron theory has been used. The third chapter includes a study of relationships between the escaping set of the composition of two transcendental entire functions and those of the individual ones. Also, we discuss certain permutable transcendental semigroups for which Eremenko's conjecture holds. In addition, we study the dynamical properties of the Fatou map,  $f(z) = z + 1 + e^{-z}$ . The Chapter 4 deals with the construction of a map  $F: \mathbb{C} \times \mathbb{R} \rightarrow \mathbb{C} \times \mathbb{R}$  with wandering domains using approximation theory. Also, we show that the transcendental skew-product,  $F(z, w) = (e^{-(z+w)} + z + w, e^{-2w} + 2w + 1)$  has a Baker domain. In Appendix, we extend the notion of a nearly abelian rational semigroup to a transcendental setting. We have also provided a class of transcendental entire functions which nearly permute with the function  $f(z) = \sin z + q(z)$ , where  $q(z)$  is a non-constant polynomial.

*Contents*

1. Introduction 2. Functions with no unbounded fatou components 3. Escaping set of composition of entire functions 4. Wandering domains and baker domains in several variables 5. A dynamic of a nearly abelian transcendental semigroup. Bibliography. Index.

15. KIPGEN (Lhinghoineng)  
**Shock Waves and the Riemann Problem for Non-Ideal Gaseous Media.**  
 Supervisor: Dr. Randheer Singh  
Th 27223

*Abstract*

In this thesis, we have considered one-dimensional quasilinear hyperbolic systems describing real dusty reacting gases, isothermal non-ideal dusty gas, and non-ideal modified Chaplygin isentropic two-phase flow consisting of non-constant source

term. The thesis consists of six chapters which is describe briefly as follows In the first chapter, we give historical background and motivation, basics definitions and method required for further chapters and a summary of the thesis. In Chapter 2, we obtained the solutions of the Riemann for shock wave, simple wave and contact discontinuities without any restriction on the magnitude of initial states. The effects of non-idealness and dust particles on the compressive and rarefaction waves are also analyzed. In Chapter 3, the transport equation for an acceleration wave is obtained to analyze the evolution of the acceleration wave via a particular solution and the amplitudes of reflected waves, transmitted waves, and the bounce in shock acceleration, which arises from the collision between an acceleration wave and a blast wave, are determined. Chapter 4 and 5, we examined the existence of  $\delta$ -shock and vacuum state in the Riemann problem for the system under consideration and it is shown that as the flux approximation and pressure vanish, the Riemann solution containing two shock waves converges to  $\delta$ -shock solution and the Riemann solution containing two rarefaction waves tends to the vacuum state solution of the transport equations. In Chapter 6, using the characteristic variables, the analytical solution of the flow field upto the first order has been carried out via perturbation method and expansion fan wave solution upto the first order are obtained. Further, the effects of dust particles, van der Waals excluded volume and reaction parameters on the velocity gradient are analyzed.

#### *Contents*

1. Introduction 2. Riemann problem for van der waals reacting gases with dust particles 3. Collision of an acceleration wave with blast wave in van der waals dusty reaction gases 4. – shocks and vacuum states in the Riemann problem for isothermal van der waals dusty gas under the flux approximation 5. Solution of Riemann problem to non-ideal modified chaplygin isentropic two-phase flow consisting of non-constant source term. 6. Velocity gradient and weak expansion fan in non- ideal dusty reacting gas via characteristic perturbation technique. Bibliography.

16. MANAS (Anany)  
**Maximality and Weaker forms of Normality in L-Groups.**  
 Supervisor: Prof. Iffat Jahan  
Th 26775

#### *Abstract*

After the pioneering work of Zadeh, the applications of fuzzy set theory are well known. In 1971, Rosenfeld developed the fuzzy subgroup. Thereafter, several concepts of group theory were generalized to the fuzzy setting. After the emergence of Head's metatheorem, the research in fuzzy algebra came to a halt. However, in the studies of L-groups, the evaluation lattice is a completely distributive lattice, hence the metatheorem is not applicable. Moreover, several limitations of fuzzy group theory are removed since the parent structure is an L-group instead of a classical group. This allowed the analogs of various notions to be adequately formulated in the L-setting. The present thesis aims to further such development in L-group theory. The thesis is divided into six chapters including first chapter as introduction. In Chapter 2, the maximal L-subgroups are formulated and their level subsets are explored. The Frattini L-subgroup is defined. The non-generators of an L-group are developed and their relation with the Frattini L-subgroup is established. The normality of the Frattini L-subgroup is investigated. In Chapter 3, the normality of maximal L-subgroups of nilpotent L-groups is explored. Several results regarding Frattini and finitely generated L-subgroups are investigated. In Chapter 4, the

conjugate of an L-subgroup by an L-point is introduced. Several properties of conjugate L-subgroups are studied. The notion of conjugacy is investigated in the context of normality and normalizer of L-subgroups. The normalizer of an L-subgroup is defined using conjugate L-subgroups. In Chapter 5, the pronormal L-subgroups are formulated. Then the pronormal L-subgroups are investigated in the context of normality and subnormality of L-subgroups. The relation of pronormality with normalizers and maximal L-subgroups is explored. In Chapter 6, the abnormal and contranormal L-subgroups are introduced. Several results are proved that establish abnormal and contranormal L-subgroups as the opposites to the notion of normality.

### *Contents*

1. Introduction 2. Maximal and frattini L-subgroups of an L-group 3. An application of maximality to nilpotent and finitely generated L-subgroups of an L-group 4. Conjugate L-subgroups of an L-group and their applications to normality and normalizer 5. Pronormal L-subgroups of an L-group. 6. Abnormal and contranormal L-subgroups of an L-group. Bibliography.

17. MANOJ KUMAR

**Mixed Convection in a Lid-Driven Square Cavity using Stream Function-Vorticity Formulation and ADI Scheme.**

Supervisors: Prof. Shobha Bagai and Prof. Arvind Patel

Th 26776

### *Abstract*

This thesis entitled “Mixed convection in a lid-driven square cavity using stream function-vorticity formulation and ADI scheme” investigates the mixed convection in a two or four-sided lid-driven square cavity using stream function-vorticity ( $\psi - \zeta$ ) formulation and ADI scheme. Mixed convection, a combination of natural and forced convection, is a mechanism of composite heat transfer. The mutual action of the buoyancy effect and shear forces causes the mixed convection. The natural or mixed convection flow with heat and mass transfer in a square cavity has a wide variety of applications such as food processing, nuclear reactors [7], cooling of electronic devices [8], float glass production [9], solar collectors [11], lake and reservoirs [12], solar ponds, crystal growth, furnaces, lubrication technologies, chemical processing equipment, and drying technologies [15], coating, melt spinning processes, oil extraction, etc. The vast array of potential uses for mixed convection flow with heat and mass transfer in a driven square cavity as mentioned above served as our inspiration for chapters 2 through 6 of the thesis. This thesis aims to investigate the mixed convection flow with heat and mass transfer in a two or four-sided lid-driven square cavity utilising stream function-vorticity formulation and ADI method. A mixed convection problem is governed by the Navier-Stokes equations, the continuity equation, the energy equation, and the mass transfer equation. It is challenging to evaluate the accuracy of the numerical solutions because there are no analytical solutions to the physically intriguing problem of mixed convection flow. The stream function-vorticity formulation is a potent strategy for resolving the two-dimensional incompressible viscous flow. This method has several advantages over other approaches for 2-D flow computations because it avoids solving the pressure field, which is one of its most interesting characteristics. This formulation does have several limitations, with the main one being that 3-D flow issues cannot be addressed with it.

*Contents*

1. Introduction 2. Heat and mass transfer in a four-sided lid driven square cavity using stream function – vorticity formulation 3. Mixed convection in a four – sided lid-driven sinusoidally heated porous cavity using stream function – vorticity formulation 4. Mixed convection in a parallel two – sided and a four – sided lid-driven square porous cavity 5. Heat and mass transfer under buoyancy effect in a two-sided anti-parallel lid driven square cavity with non-uniform sinusoidal heating on horizontal walls. 6. Heat and mass transfer under MHD mixed convection and buoyancy effect in a four –sided lid-driven square cavity. References.

18. MEENA (Om Prakash)

**Study of the Dynamics of Test Particle in the Spatial Collinear Restricted Four-Body Problem with Non-Spherical Primaries.**

Supervisor: Dr. Amit Mittal

Th 26777

*Abstract*

Celestial mechanics embraces the dynamical and mathematical theory describing the totality of possible motions of a given dynamical system. The N-body ring problem as a nonlinear dynamical system with insightful dynamics is commonly used in the modeling of spacecraft trajectory. This model is modified by including the perturbations caused by oblate spheroids and radiation pressure. This problem defines the dynamics of a test particle having negligible mass in the gravitational field of N-bodies ( $N=2, 3, 4, \dots$ ) which revolve in circular orbits about their common center of mass. The problem is investigated extensively analytically as well as numerically to understand the possible motions in the form of particular solutions along with the effect of perturbations on their motions. First, mathematical formulation of the perturbed N-body ring problem by taking radiation pressure, and with the effect of small perturbations in the Coriolis and centrifugal forces are carried out and deduced the equations of motion for the infinitesimal mass which forms a second order nonlinear differential equations. Then, equilibrium points and their stability, zero velocity curves, and fractal basins of convergence (BoC) by using the multivariate Newton-Raphson iterative method in the problem and analyzed for variations due to perturbations. A significant effect of perturbations is observed in the problem (Chapter 2). It also studied the equilibrium points and their stability, zero velocity curves, and fractal BoC in the spatial collinear restricted four-body problem (SCR4BP) by including the perturbations caused by oblate peripheral primaries and analyzed for variations due to perturbations. A significant effect of perturbations is observed in the problem (Chapter 3). Further, the variational equations of motion are determined to find the periodic motion of infinitesimal mass in the SCR4BP with non-spherical primaries. Using the Fourier series method, we have determined the long- and short-period orbits around collinear libration points. The effect of perturbing parameters on the orbital properties around the collinear equilibrium points is investigated (Chapter 4). Moreover, using the same methodology, we explored the long and short periodic orbits around the non-collinear libration points in SCR4BP with non-spherical primaries. We analyzed the effect of perturbing parameters on the orbital properties of periodic orbits.

*Contents*

1. Introduction 2. The basing of convergence associated with libration points in the perturbed  $(N+1)$ -Body ring problem 3. The spatial collinear restricted four- body problem with non-spherical primaries 4. Periodic orbits around the collinear

libration points in the spatial collinear restricted four-body problem with non-spherical primaries 5. Periodic orbits around the collinear libration points in the collinear restricted four-body problem with non-spherical primaries. References.

19. MOHD RAFIQ  
**Numerical Investigation of Certain Nonlinear Schrodinger Equations Using Wavelet Methods.**

Supervisor: Dr. Abdullah  
 Th 26778

*Abstract*

This thesis uses numerical schemes based on wavelets (Haar and Fibonacci wavelets) to find the numerical solutions of the modified unstable nonlinear Schrödinger equation and the derivative nonlinear Schrödinger equation of type-II (Chen-Lee-Liu Equation). This scheme has numerous advantages over other approaches. Overall, while many advanced numerical methods are available, the wavelets method can provide unique advantages in certain situations. This method can be particularly useful in applications such as signal processing, image analysis, and other areas where precise approximations and custom solution methods are required by offering high accuracy, efficiency, flexibility, and wavelet analysis capabilities. The entire study is divided into the following five chapters: Chapter-1: Introduction Chapter-2: Numerical Solutions of the Chen-Lee-Liu Equation Using Haar Wavelets Chapter-3: Numerical Investigation of the Modified Unstable Nonlinear Schrödinger Equation via Haar Wavelets Chapter-4: Fibonacci Wavelets Method for the Numerical Solutions of the Derivative Nonlinear Schrödinger Equation of type-II Chapter-5: Numerical Investigation using Fibonacci Wavelets Method for Solving Modified Unstable Nonlinear Schrödinger Equation

*Contents*

1. Introduction 2. Numerical solutions of the chen-lee-liu equation using haar wavelets 3. Numerical investigation of the modified unstable nonlinear schrodinger equation via haar wavelets 4. Fibonacci wavelets method for the numerical solutions of the derivative nonlinear schrodinger equation of type-II 5. Numerical investigation using Fibonacci wavelets method for solving modified unstable nonlinear schrodinger equation. Bibliography.

20. MUKESH KUMAR  
**Analysis of Phase Portrait and Resonant Curva Due to Perturbations of Geo-Synchronous Satellite Including Earth's Equatorial Ellipticity and Resistive Force.**

Supervisor: Prof. Sushil Yadav  
 Th 26779

*Abstract*

Contains the basic result and terminology to be used in the subsequent chapters. We have discussed the N-body problem, the general three-body problem, the restricted three-body problem and the restricted four-body problem. The methodology used to investigate resonant curve and phase portrait of the possible motion have been presented. Contains the investigation of resonant curves due to frequencies  $\theta_E$  (angular rate of rotation of the earth) and  $\gamma$  where  $\gamma$  is measured from minor axis of the earth's equatorial ellipse to the projection of the satellite. Perturbation technique is used to convert the non-linear equations of motion of geo-synchronous satellite to the linear form. With the help of graphs, we have shown the

effect of  $\gamma$  on oscillatory amplitude  $a$  and  $\Delta r$  (variation in  $r$ ), where  $r$  is orbital radius of satellite. By defining suitable perturbations, we have analyzed and resonant curve due to frequencies  $\theta$  (steady-state orbital angular rate of satellite) and  $\gamma$ . We have also studies the undamped and unforced phase portrait and phase space by using the method of Poincare section. Finally, we obtained energy integral  $E$  and motion of the mean longitude  $L$  for the geo- synchronous satellite.

*Contents*

1. Introduction 2. Resonant curve due to perturbations of geo-synchronous satellite including effect of earth's equatorial ellipticity 3. Resonant curve of geo-synchronous satellite including effect of earth's equatorial ellipticity and resistive force using perturbations technique 4. Analysis of resonant curve and phase portrait due to earth's equatorial ellipticity in the earth – moon system using perturbation technique 5. Analysis of resonant curve in the earth – moon system under the effect of resistive force and earth's equatorial ellipticity. 6. Analysis of resonant curve in a synchronous satellite under the gravitational effect of the sun, the moon and the earth including its oblateness using Poincare section 7. Resonant curve due to perturbations in geo-synchronous satellite including the earth's equatorial ellipticity and resistive force. Bibliography.

21. PANDEY (Atul)  
**Designing and Analysis of Public Key Cryptographic Photocols Using Certain Algebraic Structures.**  
 Supervisors: Dr. Indivar Gupta and Dr. Dhiraj Kumar Singh  
Th 27120

*Abstract*

Cryptography is the study of scientific techniques that are used to protect communications between users over an insecure channel. It provides security for the communications in terms of privacy, authenticity, non-repudiation and integrity. Cryptography can be categorized in two parts: symmetric/secret key cryptography and asymmetric/public key cryptography. In symmetric key cryptography, there is exactly one key that is shared in advance between the users who want to communicate with each other in the future. Whereas in asymmetric key cryptography, there are two keys involved namely public key and private key. These two keys are related to each other but it is computationally infeasible to extract private key from the public key.

*Contents*

1. Introduction 2. Key exchange protocol and ElGamal cryptosystem based of FDLP 3. Undeniable signature scheme on general linear group over group ring 4. Group key distribution protocol using matrices 5. Security analysis of Cryptography protocols using DLCSP over  $GL_n(F_q[S_r])$  6. Cryptanalysis of a ElGamal cryptosystem based on matrices over group rings. Conclusion and future research directions. References.

22. RAJ (Ankur)  
**Radius Problems and Construction Techniques for Univalent Harmonic Mappings.**  
 Supervisor: Dr. Sumit Nagpal  
Th 26765

*Abstract*

In complex analysis, geometric function theory is an important and an active area of research. The interplay of geometry and analysis makes this subject more fascinating. Univalent analytic functions with a proper normalization have many nice geometric properties. The Bieberbach conjecture [4] in the year 1916, was the key inspiration for researchers that lead to the rapid development of this subject. Since then, many important new ideas and methods have been emerged such as Loewner's differential equation involving slit mappings, Grunsky inequalities and logarithmic coefficients to name a few. Harmonic functions are the solutions of Laplace equation which play a crucial role in many areas of mathematics, physics and engineering. It was originally studied by differential geometers because of its connection to the minimal surfaces. The mean value property for harmonic functions has tremendous applications in mathematics and physics. Harmonic functions can also be viewed as a natural generalization of analytic functions. Due to all these perspectives, harmonic functions have become a central subject of study and research. Although the topic of harmonic functions is an old area of research but its study in view of the univalent analytic functions is a fairly recent and developing area of research. After the proof of long standing Bieberbach Conjecture in 1984 by Louis de Branges [5], many researcher shifted their interest towards generalizing the classical results in theory of univalent analytic functions for the larger class of harmonic univalent functions. It is a natural question to ask what properties of analytic univalent functions are true for harmonic univalent mappings. During the same year 1984, J. Clunie and T. Sheil- Small [10] came with an affirmative answer. In their landmark paper [10], they pointed out that many classical results for univalent analytic functions have clear analogues for univalent harmonic mappings which opened a new avenue for researchers in this field. In any simply connected domain  $\Omega$ , a harmonic function  $f$  admits a representation  $f = h + \bar{g}$ , where  $h$  and  $g$  are analytic in  $\Omega$ . Let  $H$  denotes the class of harmonic functions  $f = h + \bar{g}$  defined in the open unit disk  $D := \{z \in \mathbb{C} : |z| < 1\}$  and normalized by the conditions  $h(0) = h'(0) - 1 = g(0) = 0$ . Since the Jacobian of  $f = h + \bar{g} \in H$  is given by  $Jf(z) = |h'(z)|^2 - |g'(z)|^2$ , by a theorem of Lewy [27],  $f$  is sense-preserving in  $D$  if and only if  $|g'(z)| < |h'(z)|$  for all  $z \in D$ , or equivalently, the dilatation  $wf : D \rightarrow \mathbb{C}$  defined by  $wf(z) = g'(z)/h'(z)$  is an analytic function satisfying  $|wf(z)| < 1$  for all  $z \in D$ . Let  $H_0$  be a subfamily of  $H$  consisting of harmonic functions  $f = h + \bar{g}$  which are further normalized by  $g'(0) = 0$ . Let  $SH \subset H$  and  $SOH \subset H_0$  be subclasses consisting of sense-preserving and univalent harmonic functions. The classical family  $S$  of analytic univalent functions is a subclass of  $SOH$ . In the year 1984, Clunie and Sheil-Small [10] initiated the study of the class  $SOH$  and its subclasses  $KOH$ ,  $S^*OH$  and  $COH$  which map  $D$  onto a convex, starlike and close-to-convex domain respectively. The corresponding subclasses of  $S$  are denoted by  $K$ ,  $S^*$  and  $C$  respectively. In several instances, the properties of univalent analytic functions serve as models for generalizations to harmonic mappings. Although many classical results of analytic univalent functions have been extended for the class  $SOH$  and its geometric subclasses, but there are still several conjectures regarding the coefficient bounds and radius problems which are still unsettled. One of them is the harmonic analogue of the Bieberbach Conjecture which was proposed by Clunie and Sheil-Small [10] stating that the  $n$ th Taylor series coefficients of the analytic and co-analytic part of a function  $f \in SH$  are bounded by  $(2n^2 + 1)/3$ .

*Contents*

1. Introduction
2. Construction of stable close-to convex harmonic mappings
3. Construction of univalent harmonic mappings using a new product  $\otimes$
4. Radius

problems for the product  $\otimes$  5. Radius problems associated with the analytic part of harmonic mappings. . References. Inex.

23. SARITA KUMARI  
**Browder Spectral Theory Over the Complex and Quaternionic Hilbert Spaces.**  
 Supervisor: Dr. Preeti Dharmarha  
Th 27221

*Abstract*

The thesis, "Browder Spectral Theory over Complex and Quaternionic Hilbert Spaces" comprises seven well-structured chapters. The first chapter serves as an introduction. Chapter 2, titled "Weighted Browder spectrum" introduces the weighted Browder spectrum,  $\alpha$ -B-Fredholm,  $\alpha$ -B-Browder,  $\alpha$ -Drazin invertible operators, corresponding spectra and analyzes their properties. An example validates the non-emptiness of these classes. We investigate the relationship between  $\alpha$ -Browder operator and operators defined on invariant subspaces and characterize the weighted Browder spectrum for the sum of two bounded linear operators. Chapter 3, "Pseudo-weighted Browder spectrum" explores essential pseudo-weighted spectra and introduces  $\alpha$ -pseudo-Browder operators and their corresponding spectra, supported by an example. The investigation reveals connections between them and the stability of essential weighted and essential pseudo-weighted spectra. We establish the relationship between the pseudo-weighted spectra of operator  $T$  and operators defined on invariant subspaces and characterize the pseudo-weighted Browder spectrum for the sum of two bounded linear operators. Chapter 4, titled "Spectral properties of block operator matrices" focuses on investigating the weighted Weyl and pseudo-weighted Weyl spectra of  $2 \times 2$  block operator matrices. Further, we delve into the sequential properties of cardinal numbers and the operators defined in the previous chapters. Chapter 5, titled "Properties of Fredholm, Weyl and Jeribi essential S-spectra in a right quaternionic Hilbert space" explores Fredholm, Weyl operators and corresponding S-spectra in right quaternionic Hilbert space. We investigate the properties of Fredholm and Weyl operators, Weyl S-spectrum, spectral map property for essential and Weyl S-spectra and characterize the essential and Weyl S-spectra for the sum of two bounded linear operators. Chapter 6, titled "Browder operators on quaternionic Hilbert space" focuses on Browder operators over quaternionic Hilbert space. It explores the properties of ascent and descent, perturbation and closedness of Browder operators and corresponding S-spectra. The spectral map property for Browder S-spectrum is investigated and characterizes the Browder S-spectrum of the sum of two bounded linear operators. Chapter 7 of the thesis titled "Unanswered questions" contains some open problems.

*Contents*

1. Introduction 2. Weighted browder spectrum 3. Pseudo-weighted browder spectrum 4. Spectral properties of block operator matrices 5. Properties of fredholm, weyl and jerbi essential s-spectra in a right quaternionic hibert space. 6. Browder operators on Quaternioni hibert space 7. Unanswered questions. Bibliography. List of symbols.

24. SHARMA (Mridu)  
**Convective Heat and Mass Transfer Through Porous Media Saturated by Nanofluid.**  
 Supervisors: Prof. Shobha Bagai and Prof. Dinesh Khattar  
Th 26780

*Abstract*

The objective of this thesis is to study the unsteady two dimensional heat and mass transfer problems through a porous media for certain shaped bodies. The fluid in consideration in this thesis is a nanoparticles infused fluid called Nanofluid, which has enhanced thermal properties in comparison to the base fluid. Numerical techniques are adopted to simplify and solve the considered problems. The influence of a range of decisive parameters are studied on the skin-friction coefficient, the Nusselt number and the Sherwood number. The obtained results helps in enhancing the heat and mass transfer processes with the help of external forces. The field of convective heat and mass transfer through porous media saturated by nanofluid has been hot favourite with researchers from the past two decades, majorly due to its wide scale applications that include geothermal engineering, nuclear reactors, industrial cooling, cancer imaging and drug delivery. The disbenefits of the nanofluids are dealt in the concluding chapter by introduction of motile microorganism. The microorganisms not only enhances the stability but also a good replacement to inorganic nanoparticles as far as environment is concerned.

*Contents*

1. Introduction 2. Transient free convective heat and mass transfer for an axisymmetric body immersed in porous media saturated by nanofluid 3. Effect of viscous dissipation and radiation on transient heat and mass transfer across a permeable flat plate embedded in a porous media saturated by nanofluid 4. Transient heat and mass transfer by nanofluid past a stretching sheet saturated by porous media 5. Unsteady bioconvection of bionanofluid over a stretching sheet due to biosynthesis of microorganisms. References.

25. SINGH (Gambheer)

**Some Special Matrix Classes in linear Complementarity and their Applications.**

Supervisors: Prof. C.S. Lalitha, Prof. Samir Kumar Neogy and Prof. Promila Kumar  
Th 27226

*Abstract*

This thesis mainly deals with investigating various new matrix classes and their properties in the context of linear complementarity problem (LCP) and the extension of the class of matrices processed by Lemke's algorithm. A portion of the thesis concerns the existence and multiplicity of solutions to LCPs. Here, we highlight the importance of matrix classes in studying the LCP. A number of these matrix classes are interesting because they describe specific LCP properties. We discuss different variants of Lemke's algorithm with examples and related results. It deals with many convex optimization problems which can be transformed into a linear complementarity framework. We identify a new matrix class  $\bar{L}(d)$  and prove that this class is processable by Lemke's algorithm. An application of this new matrix class occurs in quadratic programs and polymatrix games. We have partially addressed the question of finding a proper  $d$  vector such that Lemke's algorithm successfully computes a solution. We identify two subclasses of  $Q$ -matrix, namely  $N$  and  $N^-$ -matrices of exact order  $k$  and establish sufficient conditions for these classes to satisfy  $Q$ -property. Further, we characterize strong versions of several matrix classes that appear in the LCP for uncertain data represented by intervals. Finally, we study applications of several special matrix classes arising in LCP.

*Contents*

1. Introduction 2. Different variants of lemke's algorithm and its applications 3. Some new subclasses of  $Q_0$ - matrix and their characterization 4. Matrices of exact order  $k$  5. Some special classes of interval matrices in linear complementarity theory 6. Applications of special matrix classes. Conclusions and future scope. References.

26. SINGH (Manoj Kumar)

**Distance Distribution of Constacyclic Codes Over Finite Rings.**

Supervisors: Dr. Atul Gaur and Indivar Gupta

Th 26781

*Abstract*

Hamming distance is a widely studied metric as it is more appropriate for the orthogonal modulated channel. A new coding framework for the channel whose outputs are overlapping pairs of symbols was established by Cassuto and Blaum. Such channels are known as symbol pair read channels and the distance related to this is called symbol pair distance. Recently Yakubi et al extended the framework of symbol pair read channels to the  $b$ -symbol read channels, whose output is consecutive sequences of  $b \geq 3$  pair symbols. The related distance to this is called the  $b$ -symbol distance. In this thesis, we have explored Hamming distance, symbol pair, and  $b$ -symbol distance of constacyclic codes over finite rings such as  $R = \mathbb{F}_{p^m} + u \mathbb{F}_{p^m}$ , Galois ring and Ring of Matrices. My detailed contributions are as follows: (i) We determine the hamming distances of all  $\delta$ -constacyclic codes of length  $2p^s$  over  $R = \mathbb{F}_{p^m} + u \mathbb{F}_{p^m}$ . As applications, we also determine all MDS repeated-root  $\delta$ -constacyclic codes of length  $2p^s$  over the  $R$ . (ii) We obtain symbol-pair distances of type  $(*)$ - $\delta$ -constacyclic codes of length  $2^s$  over  $GR(2^a, m)$ . We also computed the MDS symbol-pair constacyclic code of length  $2^s$  (iii) We compute the  $b$ -distance of all the constacyclic codes of length  $p^s$  over  $R = \mathbb{F}_{p^m} + u \mathbb{F}_{p^m}$  for  $1 \leq b$  and establish all MDS  $b$ -symbol constacyclic codes of length  $p^s$  over the same ring  $R$ . (iv) We derive the structure of cyclic codes of odd lengths. Moreover, we also determine the cardinality of the same cyclic codes. Furthermore, an isometric map from  $M_2(\mathbb{F}_2[u] / \langle u^k \rangle)$  to  $\mathbb{F}_4 + u \mathbb{F}_4 + \dots + u^{k-1} \mathbb{F}_4 + v(\mathbb{F}_4 + u \mathbb{F}_4 + \dots + u^{k-1} \mathbb{F}_4)$  has also been obtained which preserves their respective weight and Lee weight.

*Contents*

1. Introduction 2. Hamming of repeated constacyclic code of length  $2p^s$  over  $F_{p^m} + u F_{p^m}$  ( $u=0$ ) 3. On the symbol –pair distance of some classes of repeated – root constacyclic over galois ring 4. B-symbol distance of constacyclic codes of length  $p^s$  over  $F_{p^m} + uF_{p^m}$  5. Cyclic code over ring of matrices  $M_2(\mathbb{F}_2[u]/u^k)$ . Conclusion and future research directions.

27. SINHA (Uttam Kumar)

**Matrix-Valued Gabor Frames Over Locally Compact Abelian Groups.**

Supervisor: Prof. Lalit Kumar

Th 26782

*Abstract*

The work in the thesis entitled “Matrix-Valued Gabor Frames over Locally Compact Abelian Groups” deals with the study of frame properties of matrix-valued Gabor systems in the space  $L^2(G, C^{n \times n})$ , where  $G$  is a locally compact abelian group which is metrizable and sigma-compact, and  $n$  is a positive integer. We present an

interplay between Bessel sequences in  $L^2(G)$  and its associated matrix-valued space  $L^2(G, C^{n \times n})$ . An Aldroubi-type construction of new matrix-valued frames from given matrix-valued frames over LCA groups is given. Sufficient conditions, with explicit frame bounds, for the finite sum of matrix-valued Gabor frames to be a frame of the space  $L^2(G, C^{n \times n})$  are given. Extension of a pair of matrix-valued Gabor Bessel sequences to dual frames of the space  $L^2(G, C^{n \times n})$  is discussed. Necessary and sufficient conditions for matrix-valued  $\Theta$ -Gabor frames over LCA groups are given, where  $\Theta$  is a bounded linear operator acting on the underlying space. We provide a characterization for matrix-valued  $\Theta$ -Gabor frames which extend a result due to Găvruta. We give frame conditions in terms of bounded, linear, and adjointable (with respect to the matrix-valued inner product) operators on the space  $L^2(G, C^{n \times n})$ . We introduced the notion of matrix-valued  $(\Theta, \Theta^*)$ -Gabor frame of the space  $L^2(G, C^{n \times n})$ , where both the lower and upper frame conditions are controlled by a bounded linear operator  $\Theta$  on the underlying space. Some differences with the help of examples and counter-examples between standard Gabor frames, matrix-valued  $\Theta$ -Gabor frames and matrix-valued  $(\Theta, \Theta^*)$ -Gabor frames are discussed. We provide the existence of tight matrix-valued  $(\Theta, \Theta^*)$ -Gabor frames for  $L^2(G, C^{n \times n})$  in terms of adjointable (with respect to the matrix-valued inner product) hyponormal operators on  $L^2(G, C^{n \times n})$ . A characterization of  $(\Theta, \Theta^*)$ -Gabor frames is given. We show that matrix-valued Gabor frames,  $\Theta$ -Gabor frames, and  $(\Theta, \Theta^*)$ -Gabor frames in the space  $L^2(G, C^{n \times n})$  are stable under small perturbations.

#### *Contents*

1. Introduction 2. Matrix-valued Bessel sequences and frames over LCA groups 3. Matrix-valued  $\Theta$ -gabor frames over LCA groups 4. Matrix-valued  $\Theta$ - $\Theta^*$ -gabor frames over LCA groups. References.

28. SRIVASTAVA (Ravi Kumar)

#### **Generated Fuzzy Ideals and Applications of Metatheorem in Semigroups and Semirings.**

Supervisor: Prof. Arvind  
Th 27224

#### *Abstract*

In Chapter 1, Some definitions of classical algebra and fuzzy algebra as well as results along with some important outcomes of metatheorem is provided to make the thesis self contained. We begin Chapter 2 with some important results of metatheorem for a semigroup. We define the fuzzy quasi-ideals in semigroups. We extend the results that arise due to the generalization of interior ideals, bi-ideals and quasi-ideals to the fuzzy setting. In Chapter 3, we apply the metatheorem in a semigroup when the semigroup is either regular or a semilattice of groups. We characterize a semigroup that serves as a semilattice of left groups, semilattice of right groups, and semilattice of groups, using the notions of fuzzy left ideals, fuzzy right ideals, fuzzy ideals, fuzzy bi-ideals, fuzzy interior ideals and fuzzy quasi-ideals. In Chapter 4, we demonstrate that classes of various fuzzy ideals in semirings that possess the property of being projection closed. We investigated fuzzy bi-quasi ideals, fuzzy bi-interior ideals, fuzzy bi-quasi-interior ideals and fuzzy quasi-interior-ideals of a semiring. In Chapter 5, concept of fuzzy bi-ideals, fuzzy interior ideals and fuzzy quasi-ideals generated by a fuzzy set in a semigroup is defined. In our present work, we establish the existence of a fuzzy bi-ideal, fuzzy interior ideal and fuzzy quasi-ideal generated by a fuzzy set in a semigroup. In Chapter 6, we define level subsets and strong level subsets of a fuzzy set in a semiring. We characterize fuzzy left ideal, fuzzy right ideals, fuzzy two sided ideals, fuzzy bi-ideals, fuzzy interior ideals and

fuzzy quasi-ideals of a semigroup in terms of level subsets and strong level subsets. We obtain generated fuzzy ideals by a fuzzy set in a semiring with unity. In the Section 3 of the chapter, we obtain the same in a semiring without unity.

#### Contents

1. Introduction 2. Applications of metatheorem on fuzzy ideals of semigroups -I 3. Applications of metatheorem on fuzzy ideals of semigroups -II 4. Applications of metatheorem on fuzzy ideals of semirings 5. Generated fuzzy ideals in semigroups 6. Generated fuzzy ideals in semigroups. References.

29. SUMIT KUMAR

#### **Robe's Restricted Three-Body Problem with Finite Straight Segment Involving Viscosity Finite Straight Segment Involving Viscosity, Oblateness, Coriolis and Centrifugal Forces.**

Supervisor: Prof. Bhavneet Kaur

Th 27121

#### Abstract

Restricted Three-Body Problem (RTBP) is the problem of predicting the motion of a small body P3 which moves under the gravitational attraction of two celestial bodies P1 and P2 known as primaries. It is one of the most fascinating problem in celestial mechanics. The primaries revolve in a circular orbit about their center of mass under the influence of their mutual gravitational attraction and the mass of P3 is so small that it does not influence the motion of primaries. To study the small oscillations of the Earth's inner core under the influence of the Moon, Robe (1977) introduced a new type of RTBP which included the effect of buoyancy force. He regarded the primary P1 as a rigid spherical shell that is filled with incompressible homogeneous fluid of density  $\rho_1$  and the other primary P2 as a point mass that lies outside the shell. The third body P3 having negligible mass is a small solid sphere of density  $\rho_3$  moves inside the shell such that it does not influence the motion of the primaries. He studied the motion of P3 under two influences: (i) the attractions of the primaries (ii) the buoyancy force of the fluid. He found that the center of spherical shell is an equilibrium point of the problem which led him to deliberate on its stability. The scientific community has explored different aspects of the problem by considering different shapes of the primaries and perturbations in the Coriolis and centrifugal forces. Shrivastava and Garain (1991) explored the effects of perturbations in the Coriolis and centrifugal forces in Robe's RTBP when the densities of fluid and P3 are equal. They proved that the small perturbation in centrifugal force shifted the equilibrium point found by Robe (1977) towards the second primary P2. Plastino and Plastino (1995) revisited the Robe's RTBP by considering the shape of P1 as a Roche's ellipsoid (Chandrasekhar, 1987). They gave full treatment to the buoyancy force by considering all three components of the buoyancy force: (i) due to gravitational field of fluid, (ii) due to gravitational field of P2, and (iii) due to centrifugal force, while both the Robe (1977) and Shrivastava and Garain (1991) considered only first component of the buoyancy force. Plastino and Plastino (1995) concluded that when  $\rho_3 > \rho_1$  the equilibrium point, the center of P1, is always stable. They also pointed out connection between effect of the buoyancy force and perturbation in the Coriolis force.

#### Contents

1. Introduction 2. Effect of Finite Straight Segment on the Non-linear stability of the equilibrium point in the planar robe's restricted three-body problem 3. Effect of

viscosity on the linear stability of the equilibrium points in the robe's restricted three-body problem with finite finite straight segment 4. Perturbations in the coriolis and centrifugal forces in the robe's restricted three- body problem with finite straight segment involving viscosity 5. Effect of oblateness and viscosity on the linear stability of the equilibrium points in the robe's restricted three-body problem with finite finite straight segment 6. Perturbed robe's restricted three-body problem with finite finite straight segment involving oblateness and viscosity. Bibliography.

## 30. SWAROOP (Anand)

**Study of Ideals of L- Subrings.**

Supervisors: Prof. Iffat Jahan and Dr. Naseem Ajmal

Th 26783*Abstract*

The whole thesis is divided into six chapters. The first two chapters– Chapter 0 and Chapter1 being an introduction and preliminaries to the succeeding chapters. In the rest of four chapters, we study residual ideals, maximal ideal, prime ideals, radical of an ideal, primary decomposition of an ideal in an L-Subring. In chapter 0, a brief history of the theory fuzzy sets is provided. This chapter attempts to trace the development of the subject which put the subject matter of the thesis in its proper perspective. This chapter also presents a summary of the research work carried out in the thesis. Chapter 1, emphasizes the basis definition, results and serve a prerequisite for the research work in the thesis. In order to make the thesis self-contained, some definitions and results concerning lattices, rings and ideals are also included in this chapter. In Chapter 2, ideals of L-rings and their properties are discussed. The concept of right (left) quotient of an ideal  $\eta$  by an ideal  $\square$  of an L-ring is introduced. Most of results of quotients of ideals of ordinary ring are extended to L-ideal theory of L-ring. In Chapter 3 the concept of maximality of an ideal in an L-ring is introduced. A necessary and sufficient for an ideal to be maximal is provided. In Chapter 4, we have introduced the concepts of prime ideal, semi prime ideal, primary ideal and radical of an L-ring. These concepts provide systematic development of the theory of ideals in an L-ring. Most of the results of these concepts in classical ring theory are extended in an L-ring. We have also introduced the concepts of associated prime ideal of an ideal and irreducibility of an ideal of an Lring. In Chapter 5, we have introduced the semiprime radical and prime radical of an ideal of an L-ring. In this chapter the concept of primary decomposition of an ideal of an L-ring is introduced. An ideal of an L-ring to have primary decomposition is investigated.

*Contents*

1. Introduction 2. Preliminaries 3. Ideals and residual ideals L-subring 4. Maximal ideals of L-subring 5. Prime ideal, semiprime ideal and radical of an ideal of L-subring 6. Prime radical and primary decomposition of ideals in an L-subring. References.

## 31. SWATI ANAND

**Coefficient and Radius Estimates for Certain Subclasses of Analytic Functions.**

Supervisor: Dr. Naveen Kumar Jain

Th 26784*Abstract*

Geometric function theory is a classical branch of complex analysis which deals with geometric properties of analytic functions. The present thesis investigates certain coefficient and radius problems for analytic functions. In this thesis, we have

introduced a unified class of normalized analytic functions using differential subordination. For such functions, we proved the growth and distortion theorem. Further, we obtained sharp bounds on the initial logarithmic coefficients. In addition, we obtained bounds on initial inverse coefficients using which we determined bound on the second Hankel determinant. Next, we studied the class of Sakaguchi starlike functions and obtained bounds on second, third and fourth-order Hankel determinants and symmetric Toeplitz determinants. The bounds obtained for Toeplitz determinants are sharp. Moreover, we obtained sharp Bohr radius for certain classes of normalized analytic functions satisfying a second-order differential subordination. We also estimated sharp Bohr radius for the classes of alpha-convex functions and typically real functions. In addition, using the approach of quantum calculus we discussed the Bohr radius problem for  $q$  analogues of the classes of alpha starlike and alpha convex functions. As well we investigated the Bohr radius problem for a class with negative coefficients. Finally, we discussed the radius problem for normalized analytic functions with fixed second coefficient. We used the technique of determining the disk containing the image of the open unit disk under the mapping  $z f'(z)/f(z)$ .

#### *Contents*

1. Introduction 2. Estimates for normalized analytic functions 3. Estimates for sakaguchi starlike function 4. Bohr radius estimates for certain analytic functions 5. Bohr radius estimates using quantum calculus approach 6. Various starlikeness for analytic function with fixed second coefficient. References.

32. YADAV (Shobha)

**Approximate Controllability and Optimal Control for Stochastic differential systems with Infinite Delay.**

Supervisor: Dr. Surendra Kumar

Th 26785

#### *Abstract*

Approximate controllability and optimal control are the most prominent concepts in control theory and the research on these issues is of great significance. In this thesis, we study existence of mild solutions, approximate controllability and the existence of an optimal state-control pair for different kinds of stochastic differential systems with infinite delay. Stochastic differential equations or inclusions (SDEs or SDIs) play a crucial role in real-world problems, and they provide more accurate results than ordinary differential systems. Also, infinite-delayed stochastic differential systems describe various phenomena which arise in physical, chemical or biological processes such as the reaction-diffusion logistic equation with infinite delay. We also incorporated and discussed the theory of impulses and multivalued maps in different chapters. We utilize the theory of stochastic calculus, fundamental solution, semigroup of operators, resolvent operators, fractional power operators as well as the Hausdorff measure of non-compactness (MNC). To be more concise, the existence of mild solutions of various SDEs or SDIs is established by employing the successive approximation approach and different well-known fixed point theorems (FPTs), and the effectiveness of the developed theoretical findings has been illustrated through examples. This thesis contains six chapters. Chapter 1 serves as an introduction and form the base for the subsequent chapters. In Chapter 2 and Chapter 3, we establish the existence and uniqueness of mild solution, and the existence of optimal control for semilinear stochastic control systems with infinite delay and infinite-delayed impulsive SDEs steered by Poisson jumps, respectively. In Chapter 4, we discuss the solvability and approximate controllability of a new class of infinite-delayed SDEs driven by Poisson jumps with instantaneous and non-instantaneous impulses via the Hausdorff MNC, the Monch FPT and the resolvent

condition. Chapter 5 deals with the existence of a solution and approximate controllability of infinite-delayed SDIs via employing the resolvent condition and Martelli FPT for multivalued maps. Chapter 6 is devoted to the existence and approximate controllability of impulsive SDIs with infinite delay. Here, we utilize the theory of semigroup and the FPT due to Covitz and Nadler to derive our main results.

*Contents*

1. Introduction 2. Optimal control for impulsive SDEs with poisson jumps 3. Approximate controllability of impulsive SDEs driven by Poisson jumps 5. Approximate controllability of SDIs 6. Approximate controllability of impulsive SDIs. References.