CHAPTER 31

MATHEMATICS

Doctoral Theses

01. AJAY KUMAR
Characterising Invariant Subspaces on Some Function Spaces.
Supervisor : Prof. Dinesh Singh
Th 23913

Abstract
(Not Verified)

This thesis deals with essentially three themes distributed across three chapters with each chapter representing one theme. In Chapter 2, we first solve completely the problem of characterising the weak-star closed subspaces of BMOA that are invariant under multiplication by a finite Blaschke factor B(Z). This result generalises the main theorem of Singh and Singh, Brown and Sadek, and Sahni and Singh, which describes the invariant subspaces of the operator of multiplication by the coordinate function z on BMOA. Furthermore, we describe the common invariant subspaces of the operators of multiplication by B(z)2 and B(z)3 on BMOA thereby generalising a Theorem of Sahni and Singh which describes the common invariant subspaces of the operators of multiplication by z2 and z3. Chapter 3 deals with a complete characterisation of the invariant subspaces of TB on H1. This generalises the description of the invariant subspaces of S∗ on H1. In addition, we also present a description of the common invariant subspaces of the operators TB2 and TB3. Raghupathi has extended Beurling's theorem to the problem of invariant subspace characterisation on Lp spaces where the invariance is considered as the multiplication by each element of the subalgebra IH∞ of H∞, where I is any inner function. In Chapter 4, we prove different versions of the above-mentioned extension of Beurling's theorem in the setting of the Banach space BMOA, the Hardy spaces for uniform algebras, on compact abelian groups with ordered duals and on the Lebesgue space of the real line. In the case of uniform algebras, we also present an elementary proof of Beurling's theorem. We also give a new proof of the Helson-Lowdenslager version of Beurling's theorem on compact abelian groups.

Contents

02. ARORA (Rajni)
A Class of Stable Numerical Approximations for the Solution of Second Order Hyperbolic Partial Differential Equations.
Supervisor : Dr. Swarn Singh
Th 23915

Abstract
(Verified)

The main objective of the present thesis is to present a class of stable numerical approximations for the solution of second order hyperbolic partial differential

equations subject to appropriate boundary conditions. This thesis contains six chapters. Chapter 1 includes some basic concepts. Chapter 2 entitled "Highly accurate unconditionally stable methods for the solution of one-dimensional hyperbolic equation with Neumann boundary conditions" is based on Numerov type discretization and application of Richardson extrapolation. Details of appropriate treatment of Neumann boundary conditions are discussed. In Chapter 3, entitled "Unconditionally stable methods for the solution of two- and three-dimensional hyperbolic equation with Robin boundary conditions", Numerov type discretizations for two- and three-dimensional hyperbolic equations are discussed. Appropriate treatment of boundary conditions is done. Corresponding alternate direction implicit methods are also discussed. Chapter 4 entitled "Numerical method based on exponential B-spline collocation method for one dimensional hyperbolic equation with Dirichlet boundary conditions" includes the details of exponential B-splines collocation method applied to a non-linear hyperbolic equation. As a special case of exponential B-splines, cubic B-splines are also used in numerical examples. Chapter 5 entitled "Numerical methods based on exponential B-spline collocation method for one dimensional hyperbolic equation with Neumann boundary conditions" includes two methods. In first, time variable is handled by using finite differences approach and in second, by strong stability preserving Runge Kutta method. Chapter 6 entitled "Numerical methods based on modified B-Splines for two-dimensional hyperbolic equation" is an extension of method discussed in chapter 4 to the corresponding two-dimensional non-linear problem. Bi-cubic B-splines are also discussed as a special case of two-dimensional exponential B-splines. The methods discussed in Chapters 2-6 have been proved to be stable by matrix stability analysis method and numerical experiments have been carried out in order to prove the efficiency and accuracy of the methods.

Contents

03. CHAUHAN (Harsh Vardhan Singh)
Connectedness, Separation Axioms and Generalized Closed Sets in Generalized Topological Spaces.
Supervisor : Dr. B. K. Tyagi
Th 24302

Abstract
(Verified)

The thesis deals with the study of generalized topological spaces and investigates several types of separation axioms, connectedness and generalized closed sets. It consists of seven chapters. Chapter 1 discusses historical developments and basic

concepts and notions of generalized topological spaces. We have tried to include enough basic concepts here to make the thesis self-contained. In Chapter 2, we introduce several separation axioms, in the framework of generalized topological spaces. The two types of initial generalized topologies are introduced. It is shown that most of these separation properties can be carried to these initial generalized topologies. Chapter 3 deals with the notion of semi-open sets and feebly open sets in generalized topological spaces. Several properties of these notions are discussed. Also this chapter considers (semi and feebly)-separation axioms for generalized topological spaces. We further investigate (semi-continuous, feebly-continuous, almost open)-functions in generalized topological spaces. In Chapter 4, following the same approach as in chapter 2, we introduce the definition of connectedness in generalized topological spaces and introduce a weak connectedness in generalized topological spaces called cμ-cμ-connectedness. The images and preimages of μ-connected sets under continuous mapping are investigated. Chapter 5 deals with more general form of extremally μ-disconnected generalized topological spaces and its properties are studied. We have shown that in an extremally μ-disconnected GTS, different notions of connectedness coincide. In Chapter 6 the γθ-operator introduced by W. K. Min is used to extend the notion of θ-connectedness to GTSs. A form of connectedness, called V-θ-connectedness is introduced and is shown to be weaker than θ-connectedness and stronger than cμ-cμ-connectedness. Chapter 7 covers several types of generalized closed sets in GTSs and their relationships with each other. An axiom between μ-T0 and μ-T1 called μ-T1/2 is introduced. Equivalent conditions are provided for μ-regularity, μ-normality and extremally μ-disconnectedness.

Contents

04. AYSHA IBRAHEEM

Multi-switching Synchronization of Chaotic and Hyperchaotic Dynamical Systems.

Supervisors : Dr. Mridula Budhraja and Prof. Ayub Khan

Th 24301

Abstract
(Not Verified)

The aim of this thesis is to develop new multi-switching synchronization schemes for multiple master-slave systems and generalization of the existing multi-switching synchronization scheme for one pair of master and slave systems. Chapter 1 is introductory, which contains the history of chaos synchronization, basic concepts which are required in the subsequent chapters, previous existing synchronization schemes, techniques for obtaining chaos synchronization etc. Chapter 2 provides multi-switching synchronization of two non-identical chaotic systems for fully unknown parameters. A generalized scheme is presented to obtain different types of multi-switching synchronization, such that the existing scheme of complete multi-switching synchronization may be obtained as a particular case of the presented approach. Numerical results are performed to validate the theoretical analysis. Chapter 3 presents an extension of combination synchronization of two master and one slave chaotic systems to three master and one slave hyperchaotic systems using nonlinear control and Lyapunov stability theory. Multi-switching

combination synchronization of two master and one slave system is achieved as a particular case. In Chapter 4, compound synchronization scheme involving one scaling master system is combined with multi-switching for four non-identical chaotic systems. An active backstepping method is employed to achieve the desired synchronization. Multi-switching combination synchronization is obtained as a particular case of the presented scheme. Chapter 5 presents synchronization for three pairs of master systems and one pair of slave system. By choosing appropriate scaling matrices and state vectors, the presented scheme can be reduced to different synchronization schemes such as multi-switching compound synchronization, multi-switching combination synchronization, etc. Chapter 6 presents a synchronization scheme of five master and one slave chaotic systems which is first of its kind. In this approach, a compound scaling signal is constructed of three master systems and then it is applied to two base master systems in multi-switching manner.

Contents

05. JAIN (Deepti)
2-Point Set Domination and Graphoidal Domination in Graphs.
Supervisors : Dr. Purnima Gupta and Dr. Anupama Panigrahi
Th 24266

Abstract
(Not Verified)

Graph theory is an area that has developed into a very broad research field since its introduction by Euler in 18th century. Application of graph theory is well known in chemistry, physics, computer science, communication science, computer technology, sociology and now several challenging problems of bio-informatics are being tackled by techniques in graphs and combinatorics. The word “domination” is quite common in our daily usage. What mathematical formulations and significance this notion of “domination” could have? This question has been gaining momentum these days in the field of discrete mathematics. Several shades of this notion have been formulated during the last three decades and the number of research papers containing results of investigation in the respective directions have become so large that it is impossible to quote here even the most significant ones amongst them.The notion of domination has now emerged as one of the major theories in graph theory, with the publication of three massive monographs exclusively covering major developments in the theory of dominations in graphs. Acharya and Sampathkumar in 1987 introduced the concept of graphoidal covers for finite graphs as a close variant of another emerging discrete structure called semigraphs. The notion of graphoidal covers arises in a natural way in a number of real life situations. In 1999, Acharya and Gupta extended the concept to infinite graphs and introduced notion of domination in graphoidally covered graphs. This initiation provides a fresh ground for further research in newer directions of domination theory. This thesis demonstrates that there is a vast scope of research in newer directions of the theory of domination to graphoidally covered graphs. The thesis mainly deals in two variation of domination namely; 2-point set domination in graphs and domination in graphoidally covered graphs.

Contents

1. Introduction 2. 2-Point set domination in graphs 3. Minimal 2-point set dominating sets in graphs 4. Independent 2-point set domination in graphs 5. Global 2-point set domination in graphs 6. Domination in graphically covered graphs. Bibliography.

06. JYOTI
Optimization Techniques for Mathematical Programming Problems.
Supervisor : Dr. Promila Kumar
Th 23918

Abstract
(Verified)

Chapter 1 contains preliminaries and a brief survey of relevant literature to the present work. A brief summary of the work done has been presented. Chapter 2 presents efficiency of order m through higher order invexity for Multi- Objective Semi-Infinite Programming Problem. Section 2.1 deals with Non-Differentiable Multi-Objective Semi-Infinite Programming Problem. Concept of efficiency and weak efficiency of order m is defined. The notion of ρ-invexity of order m has been introduced. Sufficient optimality and duality results are established. In Section 2.2, Multi-Objective Semi-Infinite Variational Problem has been explored. Necessary and sufficient optimality conditions are established. Two duals are proposed and relationship between primal and dual problems are established by means of weak and strong duality theorems under the assumptions of ρ-invexity of higher order. In Chapter 3, focus is on Multi-Objective Fractional Variational Problem. Notion of F-Kuhn Tucker point and F-Fritz John point was introduced in Section 3.1. Parametric dual is proposed and various duality results are proved under the assumptions of F-KT as well as F-FJ pseudo-invexity. We undertake Multi-Objective Semi-Infinite Fractional Variational Problem in Section 3.2. Notion of invexity and generalized invexity have been defined. Necessary and sufficient optimality conditions are established. A parametric dual is proposed and relationship between primal and dual is established. Chapter 4 deals with mathematical programming problem with infinite constraints. Section 4.1 introduces Non-Smooth Semi-Infinite Interval Valued Programming Problem. LU-optimal solution concept is defined. Necessary optimality conditions are derived using concept of LU-optimal solution. Sufficient optimality conditions are proved under the assumption of invexity. Two duals are proposed and duality theorems are established. In Section 4.2, notion of higher order B-type I function is introduced for Multi-Objective Semi-Infinite Variational Problem. Concept of efficiency is used as a tool for optimization. Optimality conditions and duality results are established.

Contents

1. Introduction 2. Efficiency of order *m* through higher order invexity for semi-infinite programming problem 3. Variational problems with fractional objective 4. Mathematical programming with infinite constraints .Bibliography.

07. GANDHI (Shweta)
Subordination for Starlike Functions.
Supervisor : Prof. V. Ravichandran
Th 23919

Abstract
(Verified)

The main objective of the thesis is to discuss geometric properties of certain subclasses of star like functions defined in the open unit disk. In chapter 1, brief survey of the basic concepts and results from the classical theory of univalent functions are discussed, that are relevant to our further investigation. In chapter 2, subordination results for Carath\'eodory functions are investigated. In addition to this sufficient conditions are obtained for normalized analytic functions to belong to subclasses of star like function. In chapter 3, we determine radius estimates, coefficient estimates for the class of star like functions associated with lune and three leaf function. In chapter 4, geometric and analytical properties of convex combination of exponential and linear function is studied thoroughly. In chapter 5, a subclass of meromorphic star like functions with negative coefficients is introduced. We obtain coefficient inequalities, growth and distortion inequalities, representation theorem and closure properties for the new class introduced in the chapter.

Contents

08. MAHESH KUMAR
Asymptotics of Composition Operators on Banach Spaces of Holomorphic Functions.
Supervisor : Dr. Sachi Srivastava
Th 23917

Abstract
(Verified)

We study the asymptotics of composition operators on Banach spaces of holomorphic functions on Ω, where Ω=D or C+, where D is the open unit disk and C+ is the right open half of the complex plane. It turns out that for composition operators on several Banach spaces of holomorphic functions on D, including the Hardy spaces Hp (D) and the standard weighted Bergman spaces Aβp (D), $1\leq p<\infty$, $\beta>-1$, the situation is very special : if the powers of a composition operator Cφ converge strongly, then they converge uniformly. For Ω=C+, we show that on Hardy spaces Hp (C+), $1\leq p<\infty$ and standard weighted Bergman spaces Aβ2(C+), $\beta>-1$, when $\varphi:C+\rightarrow C+$ induces a bounded composition operator Cφ on these spaces, and φ is not of parabolic type, then either the powers of Cφ converge uniformly only to 0 or they do not converge even strongly. We further show that this is equivalent to having the Denjoy-Wolff point of φ at infinity with $0<\varphi'(\infty)<1$. We also obtain convergence results for composition operators Aβp (C+), $\beta>-1$, $1\leq p<\infty$ for special φ's. We note that there are maps of parabolic type for which the powers of Cφ converge strongly to 0, but the powers does not converge uniformly. Finally, we apply all our results to study the asymptotic behaviour of semigroups of composition operators on several Banach spaces of holomorphic functions associated with holomorphic semiflows on Ω. Our results rely on a good knowledge of the essential spectrum of Cφ and the location and behaviour of the Denjoy-Wolff point of φ, expressed by the Denjoy-Wolff theorem. It also depends on the beautiful essential spectral radius formulae that we establish respectively, for composition operators on Hardy spaces over C+, and standard weighted Bergman spaces over D and for a special φ over C+.

Contents

1. Introduction 2. Banach spaces of holomorphic functions 3. Boundedness and power boundedness of Composition operators on X 4. Essential spectral radius formulate for Composition operators on Hardy spaces and standard weighted Bergman spaces. Simple ploes and uniform convergence 6. Asymptotics of Composition operators on $X \rightarrow$ Hol(Ω), where Ω = D or C_+ 7.Semigroups of Composition operators on $X \rightarrow$ Hol (Ω), where Ω = D or C_+ .References and Index.

09. MALHOTRA (Nidhi)
Some Fixed Point Theorems in b-**Metric,** b-**Metric Like and** G-**Metric Spaces.**
Supervisor : Dr. Bindu Bansal
Th 23911

Abstract
(Verified)

In the present thesis, we have studied b-metric, b-metric-like and G-metric spaces. On b- metric spaces, we have proved various common fixed point theorems, coincidence point theorems and common coupled fixed point theorems under diverse conditions. Reich- type multivalued contraction on a complete b-metric space endowed with a graph has also been discussed. Further, Suzuki-type fixed point theorem has been generalized in the framework of b-metric-like spaces. Finally, we have defined and studied various generalization of F-contraction in G-metric spaces. The thesis is organised into seven chapters followed by References, of which Chapter 1 is preliminary in nature wherein we have reviewed historical development along with recent trends in fixed point theory. Chapter 2 consists of fundamental notations and terminologies used frequently in succeeding chapters. In chapter 3, we have proved a common fixed point theorem for six commuting and weakly compatible mappings and established common coupled fixed point theorems for contractions characterized by certain generalized rational expressions in b-metric spaces. In chapter 4 we defined the notion of cyclic (α,β)-(ξ,ψ,φ)-contractive mappings and established the existence of coincidence point and common fixed point for a hybrid pair and a pair of single valued mappings in b- metric spaces. In chapter 5, we initiated the study of a new class of Reich type multi-valued contraction and established some results on a complete b-metric space endowed with a graph. Chapter 6 is devoted to extension and generalization of Suzuki-type fixed point theorem in the setup of b- metric-like spaces. Chapter 7 deals with introduction and study of various generalizations of F-contraction in the setting of G-metric spaces.

Contents

1. Historical background 2. Prerequisites 3. Some common and coupled fixed point theorems on b-metric spaces 4. Cyclic admissible contractions in b-metric spaces 5. b-Riech type weak contraction on b-metric spaces endowed with a graph 6. Suzuki-type fixed point theorems in b-metric like spaces 7. Some new type of contractions on G-metric spaces .References.

10. MANOJ KUMAR
Design and Analysis of Symmetric Cryptographic Primitives.
Supervisors : Dr. Anupama Panigrahi and Dr. S. K. Pal
Th 23910

Abstract
(Not Verified)

Cryptography deals with the security aspects of communication in the presence of an adversary. This comprises the study of mathematical techniques used in information security such as confidentiality, authenticity and data integrity. Block ciphers have a long history and turn out as the most widely used cryptographic primitives. A block cipher can be used to build other cryptographic primitives like stream ciphers and hash functions. The thesis is motivated from the widespread applications of symmetric cryptography and especially of lightweight block ciphers in RFID tags, sensor networks and cloud computing. We propose a lightweight block cipher which provides good security and better performance in software implementations. This is used to design a hash function with better efficiency in software applications. Permutation layer adequately mixes the input data with key bits and plays an important role in the security of a scheme. We propose numerous candidates for diffusion layer which can be used to design the secure and efficient block ciphers. Branch-and-bound based algorithm is used to search the differential trails in block ciphers. We analyse two lightweight block ciphers and provide optimal differential trails to improve the security bounds of these ciphers. In Chapter 1, we provide background, cryptanalysis methods and motivation for designing the secure and efficient cryptographic primitives. In Chapter 2, we propose lightweight block cipher FeW by using an admixture of Feistel and generalised Feistel structures and provide its security analysis from various cryptanalysis attacks. Chapter 3 proposes a hash function HeW based on FeW. It provides security analysis of HeW and compares its efficiency with other hash function. Chapter 4 proposes several candidates for a diffusion layer with 16/32-bit input size which can be used to design new block ciphers. Chapter 5 provides the accurate bounds for differential cryptanalysis of lightweight block ciphers ANU and PICO.

Contents

11. RAJEEV KUMAR
Cryptography using Elliptic Curve Groups and Pairings.
Supervisor : Dr. Arvind and Dr. S. K. Pal
Th 23912

Abstract
(Verified)

The present thesis deals with the mathematics of elliptic curve groups and pairings on elliptic curve groups. We introduce some complex problems by using elliptic curve groups and pairings. We also construct some efficient and secure cryptographic schemes using elliptic curve groups and pairings. The thesis consists of five chapters. Chapter 1 discusses historical developments and basic results of elliptic curves. It also discusses mathematical concepts used in cryptography. In Chapter 2, we give arithmetic of pairings on elliptic curve groups. We construct the self pairings on some super singular elliptic curves. We provide some elliptic curve groups which are suitable for the computation of pairings. We also prove that conventional classification of pairings into Type 1, 2, 3 and 4 is applicable for pairing-friendly elliptic curves of embedding degree one proposed by Wang et al. Chapter 3 deals with elliptic curve based key agreement schemes. In this chapter we present the concept

of Asymmetric Group Key Agreement (ASGKA) introduced by Wu et al. and extend the work of Zhang et al. by constructing a more efficient and secure elliptic curve based ASGKA scheme from Tate pairing. In Chapter 4, first we give security analysis of Chaudhry et al.'s authenticated encryption scheme. To overcome the weaknesses of Chaudhry et al.'s scheme we also give improved authenticated encryption scheme. We also propose an e-payment system using our proposed encryption scheme and carry out subsequent analysis. Chapter 5 deals with elliptic curve based signature schemes. We use our self pairing to construct a digital signature scheme. We also construct a new short signature scheme with the same features as of ZSS but from a different pairing on elliptic curve groups. Our construction takes advantage of using Type 3 pairings, which are widely used for high security and efficiency reasons.

Contents

1. Introduction 2. Pairing and pairing-friendly elliptic curves 3. Elliptic curves based key agreement schemes 4. Encryption and authenticated encryption using elliptic curves 5. Elliptic curve based digital signatures .References.

12. PRAJAPATI (Nitish)
Multi Switching Synchronization of Chaotic Multiple Drive-Response Systems.
Supervisor : Dr. D Khattar
Th 23909

Abstract
(Not Verified)

Our research work is based on the study of Synchronization in Chaotic dynamical systems. A response chaotic dynamical system under the influence of an external controller is said to synchronize with a drive chaotic dynamical system when the difference/sum between their state vectors, which is called synchronization error, converges to zero asymptotically. There are two things which are crucial to understand a synchronization problem. Firstly, formulation of synchronization error as it gives the direction in which the synchronization problem proceeds. For instance, if instead of the difference between state vectors, we choose synchronization error as the sum of the state vectors, then we achieve what is called as anti synchronization of chaotic systems. Similarly, by formulating synchronization error in different ways we get different kind of synchronization schemes. Secondly, synchronization happens under the influence of an external controller, that is, we have to design a suitable controller for which we achieve the desired synchronization. These studies on synchronization of chaotic systems are of great importance and interest because chaotic systems intrinsically defy synchronization because of the well known property of chaotic systems known as sensitive dependence on initial conditions. Because of this property, even two identical systems starting from slightly different initial conditions, will evolve in time in an unsynchronized manner. In this thesis, we have specifically focused our efforts on extending multi switching synchronization to multiple drive-response systems. This is an almost untouched area of research and the idea was recently initiated by Vincent et al. Motivated by this idea, in our work we combine multi switching synchronization scheme with different synchronization schemes concerning multiple drive response systems and introduce some new synchronization schemes.

Contents

1. Introduction 2. Multi switching combination synchronization of systems with unknown parameters 3. Multi switching compound anti synchronization of four

chaotic systems 4. Multi switching combination-combination synchtonization of chaotic systems 5. Multi switching dual combination-combination synchronization and anti synchronization of eight chaotic systems .Index.

13. RASTOGI (Shard)
Operator Semigroups for Delay Differential Equations.
Supervisor : Dr. Sachi Srivastava
Th 24303

Abstract
(Not Verified)

Delay differential equations appear in the study of several physical and biological processes, for example, age population equation with delay. In this thesis we study some properties of the solutions of such equations by employing the theory of strongly continuous semigroups, following the framework provided by Batkai et al. In particular, we look at stability, quasi-hyperbolicity of delay semigroups and study some non-analytic growth and spectral bounds associated with such semigroups. We study the case of bounded delay as well as unbounded delay. We give conditions under which orbits of the delay semigroup decay to zero, as t tends to infinity and look at `polynomial' decay rates. The behaviour of strong and polynomial stability under relatively bounded perturbations is also examined. The relationship between spectral bounds and non-analytic growth bounds of the generator of strongly continuous semigroup and that of the associated delay semigroup has been studied. We also investigate the behaviour of the non-analytic growth bound of a strongly continuous semigroup under relatively bounded perturbation. An investigation of quasi-hyperbolicity of the delay semigroup associated with the delay differential equation has been done on a Hilbert space and a Banach space separately. The behaviour of quasi-hyperbolicity of strongly continuous semigroup under relatively bounded perturbation has also been studied. Finally, we investigate the behaviour of strong and polynomial stability of the strongly continuous semigroup under unbounded delay operator. We study spectral bounds and non-analytic growth bounds under unbounded delay operators.

Contents

14. SHARMA (Jyoti)
Uncertainty Principles on Locally Compact Groups.
Supervisor : Prof. Ajay Kumar
Th 24304

Abstract
(Not Verified)

The thesis explores various uncertainty principles in particular, qualitative uncertainty principle(QUP), Hardy's theorem, Beurling's theorem for Gabor transform

and Heisenberg type uncertainty inequality for wavelet transform on locally compact groups. It is shown that QUP hold for several low dimensional nilpotent Lie groups including Heisenberg group. The conditions under which QUP hold on groups having a subgroup of finite index, are obtained. A weaker version of QUP on Moore groups is also discussed. QUP for Gabor transform on the groups of the form H×D, where D is a discrete group has been investigated. It is proved that Hardy's theorem fails for a connected nilpotent Lie group G which admits a square integrable irreducible representation. On the positive side, Hardy's theorem for groups of the form $\mathbb{R}$ n×K, where K is a compact group is established. Hardy's type theorem for Fourier transform on connected nilpotent Lie groups having non-compact center has been discussed. Beurling's theorem for Gabor transform on locally compact abelian groups with non-compact connected component and groups of the form $\mathbb{R}$ n×K, where K is a compact group, has been proved. The support of wavelet transform associated with a square integrable irreducible representation of a homogeneous space is shown to have infinite measure. It is established that every pair of admissible wavelets possesses homogeneous approximation property for wavelet transform. Moreover, an analogue of Heisenberg type inequality has been obtained for wavelet transform on wavelet groups of the form B $\ltimes$ A, where B and A are second countable, unimodular, locally compact groups of type I.

Contents

15. SHARMA (Megha)
Vector Optimization Problems Involving Vector and Set-valued Maps.
Supervisors : Dr. Dr. S. K. Suneja and Prof. C. S. Lalitha
Th 23916

Abstract
(Verified)

The purpose of the thesis is to study optimality, duality and saddle point results for certain vector optimization problems involving vector-valued or set-valued maps in their objectives and constraints. The content of thesis is divided into four chapters. Chapter 1 is introductory which is further divided into three sections. Chapter 2 is devoted to the study of optimality and duality results for vector optimization problems over cones. This chapter is further divided into two sections. Section 2.1 deals with optimality and duality results for vector optimization problems over cones using generalized arc wise cone connected d-type-I functions. In Section 2.2, we study approximate optimality and duality results for vector optimization problem over cones. The objective of Chapter 3 is to study approximate efficiencies for set-valued optimization problem and prove results related to them. This chapter is divided into two sections. In Section 3.1, we introduce the notion of approximate efficiency which refine the concept of proper efficiency and derive scalarization and approximate Lagrangian multiplier theorems for this problem. In Section 3.2, we define approximate efficiency and obtain approximate optimality and duality results. In Chapter 4 we study second-order optimality and duality results for vector and fractional optimization problems which deal with optimizing vector-valued objective functions with respect to a partial order induced by nonnegative orthant subject to certain constraints. This chapter is divided into two sections. In Section 4.1, we introduce second-order right differential of a vector-valued function with respect to an arc which is used to define new classes of generalized second-order arc wise

connected, semi-pseudo and semi-quasi arc wise connected functions. Using these functions we obtain second-order optimality and duality results for vector optimization problem. In Section 4.2, we derive second-order optimality and duality results for fractional optimization problem.

Contents

1. Introduction 2. Vector optimization problems over cones 3. Set-valued optimization problems over cones 4. Vector and fractional optimization problems .Scope for further research .Bibliography.

16. SHARMA (Sachin)
High Accutacy Numarical Methods Based on Spline Approximations for 1-D Quasilinear Parabolic and Time-Dependent Biharmonic Equations.
Supervisors : Dr. Swarn Singh and Prof. R. K. Mohanty
Th 24264

Abstract
(Verified)

The present thesis investigates the one-dimensional quasilinear parabolic and time-dependent biharmonic equations by using various spline techniques. In Chapter 1, an introduction to the basic concepts of matrix theory in relation to numerical solution of one-dimensional quasilinear parabolic and biharmonic equations has been given. In Chapter 2, we construct a new two-level implicit cubic spline method of fourth order accuracy for the numerical solution of 1-D quasilinear parabolic PDE. We have also discussed the numerical method based on graded mesh. The method can be extended in vector form to solve the system of quasilinear parabolic equations. In Chapter 3, we design a new method based on spline in compression approximations for the numerical solution of 1D quasilinear parabolic equation. In Chapter 4, we derived a numerical scheme based on spline in tension approximation for the solution of 1D quasilinear parabolic PDE. We implement the methods discussed in Chapters 2, 3 and 4 on various real world parabolic problems such as generalized Burgers-Huxley equation and generalized Burgers-Fisher equation. In Chapter 5, we construct a new cubic spline method for time-dependent biharmonic equations. In this approach, we have reduced the fourth-order PDE to a coupled system of two second-order equations. In a similar manner, we develop a two-level implicit method based on spline in compression for time dependent biharmonic problems in Chapter 6. In Chapter 7, we present a numerical algorithm based on spline in tension approximations for biharmonic problems. To demonstrate the strength and utility of the proposed method, we have solved well-known physical problems of real world in Chapters 5, 6 and 7.

Content

1. Introduction 2. Hingh accuracy numerical method based on off-step cubic spline approximations for the system of 1D quasilinear parabolic partial differential equations 3. High accuracy numerical method for the system of 1D quasilinear parabolic partial differential equations based on off-setup spline in compression approximations 4. High accuracy numerical method based on off-setup spline in tension approximations for the system of 1D quasilinear parabolic equations 5. A new two-level implicit scheme based on cubic spline approximations for the unsteady 1D quasilinear biharmonic problems 6. A new two-level implicit scheme based on non-polynomial spline in compression approximations for the unsteady

1Dquasilinear biharmonic problems 7. A new two-level implicit scheme based on non-polynomial spline in tension approximations for the unsteady 1D quasilinear biharmonic problem .Scope for future research work and Refrences.

17. SINGH (Rajesh)
Domination, Graphoidal Covers and Graphoidal Length in Graphs.
Supervisors : Dr. Purnima Gupta and Dr. Anupama Panigrahi
Th 24263

Content

1. Introduction 2. Domination and independent domination 3. Point-set domination in graphs 4. Domination in graphoidally covered graphs 5. Truly non-trivial graphoidal covers of a graph 6. Graphoidal length of a graph .Bibliography.

18. SINGH (Shivam Kumar)
Toeplitz and H-toeplitz Operators on Spaces of Analytic Functions
Supervisor : Dr. Anuradha Gupta
Th 23914

Abstract
(Verified)

The intent of this thesis is to explore and study Toeplitz and H-Toeplitz operators on the spaces of analytic functions. The necessary conditions for the hyponormality of Toeplitz operators on the Fock space are obtained. As an extension of weighted composition operator on the Fock space $\mathbb{F}^2$, the notion of Toeplitz composition operator on $\mathbb{F}^2$ is introduced. The necessary and sufficient conditions for the boundedness and compactness of Toeplitz composition operators are obtained on $\mathbb{F}^2$. The thesis also extends the study of Toeplitz operators on $\mathbb{F}^2$ to the class of slant Toeplitz operators on the Fock space. The study of k^{th}-order slant weighted Toeplitz operators also extended to k^{th}–order essentially slant weighted Toeplitz operator on weighted space $L^2(\beta)$ and $H^2(\beta)$. The study of HToeplitz operators on H^2 is extended by the class of slant H-Toeplitz operators on the Hardy space H^2 and its characterizations are obtained. The notion of H-Toeplitz operators on the Bergman space $L_a^2(D)$ is also studied. Moreover, in order to study the adjoint of multiplication operators on the derivative Hardy space $S^2(D)$, the notion of Toeplitz type operators on $S^2(D)$ is introduced.

Contents

1. Introduction 2. Toeplitz operators on the fock space 3. Generalized slant Toeplitz operators 4. H-toeplitz operators 5. Toeplitz type operators on the derivative Hardy space .References.